BODMAS

What does it mean?

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This is NOT an explanation for beginners. It is an account of some of the differences that have been found in the meaning given to the second letter. This all began with a casual discussion about algebra, during which something came up about what the place of BODMAS and what exactly it stood for. Later, a search through some relevant material made it apparent that there were different interpretations of the **O**. *This is a summary of those differences.*

The starting position is this

B is for **Brackets**

O is for ?

D is for **Division**

M is for **Multiplication**

A is for **Addition**

S is for **Subtraction**

which is merely a mnemonic to assign the orders of precedence in dealing with the operations contained in a mathematical statement. Here we deal only with the **O**.

There seem to be 5 cases.

1. It has **NO** meaning.

This is found where it is written in full, but the subsequent text goes on to deal only with the B, D, M, A and S.
The implication is that the real mnenonic is BDMAS and the O is only put in to make it pronounceable. (*This would be an acronymic mnemonic.*)
In this case it might be better written as BoDMAS.

2. It means **OFF**

This is where the first two letters are taken together to mean "Brackets Off", implying that the first thing to be done is to work out everything which is inside any brackets that are present, so that they can be removed. Or that the contents of all the brackets must be worked out first.

The author has a fond memory of this one connected to a lecturer who, when setting out to simplify an expression, always started off by referring to "Shake out the brackets". This generated a never-forgotten image of taking hold of the ends of the expression, picking it up, shaking it vigorously, and watching the brackets fall to the floor!

3. It means **OF**

This is a reference to the usage in connection with fractions like $\frac{1}{2}$ ***of*** $2\frac{3}{4}$
Since students should already know that 'of' represents × in this instance, it would seem to be unnecessary to give it a place in BODMAS, except that it does indicate a multiplication which has a priority over the Division needed in the fraction which follows. On the other hand, 'of' is not usually seen in the sort of mathematical expressions for which BODMAS needs to be invoked.
Some writers say that 'of' is a contraction for 'to the power **of**' (= meaning 5)

4. It means **OVER**

This again is a reference to fractions, only now they are of the algebraic variety, and it means that the fraction must be evaluated before the overall statement can be dealt with. An illustrative context would be something like

$$(a+b)\times\frac{5a-b}{2(7a+9)}$$

So it is a reminder that the division implied by the horizontal line of the fraction has to be done after the contents within the brackets have been dealt with. The need for this becomes even more apparent when the whole thing is written as a single in-line statement

$$(a+b)\times((5a-b)\div(2(7a+9)))$$

and also shows how much quicker the first version is to comprehend at a glance, and why it is the 'preferred' form.

Certainly this is the meaning the author was given at school, a long time ago.

It might be worth mentioning that the line itself is a form of bracket, and was used as such in the past, which could be used to explain why we are required to work out the values of the statements above and below it before attempting to divide the top by the bottom. Not exactly true, but close enough. (*see* 'vinculum' *in the notes*)

5. It means **ORDER**

This is an oddity. The explanation usually offered is something like – this is an old word meaning 'index' or 'exponent' or 'power' – a 'fact' for which no supporting evidence has been found. Why not just use 'index' or 'exponent' to give BIDMAS or BEDMAS? These are in fact now being seen more often, usually in the form of the first named.
So BIDMAS is used to prevent $4a^2$ being treated as $(4a)^2$ which is certainly a common error in students' work
But is it really needed? The individual facing up to evaluate $4a^2bc^3$ must surely have to know that the indices are merely shorthand notation, that the whole thing is a contraction of $4\times a\times a\times b\times c\times c\times c$?
But then remember that indices can be negative and/or fractional.

The **Vinculum** and **Brackets**

These are known in mathematics as ‘signs of aggregation’. They are used to indicate a part of an expression which is to be treated as an entity and evaluated by itself before being incorporated into the complete expression.

Such signs were needed as algebra became more dependent on symbols, a development which took place during the 15th and 16th centuries. Earlier algera which used words much more (usually abbreviated) did not need such a sign.

At first every mathematician devised their own sign of aggregation.

The **vinculum** *(meaning ‘to bind’)* was the first of these signs to become more generally used. It was just a straight line drawn over (or sometimes under) the terms that were to be grouped together.

For example, what we would now write as $5(x + y)$ was written as $5\ \overline{x + y}$

Brackets can be found in very early use too, but only rarely, and they did not come into general use until the 18th century. The four types most commonly found are

(round brackets or parentheses **)**
[square brackets **]**
{ curly brackets or braces **}**
⟨ angle brackets **⟩**

Brackets are always used in pairs, and the use of different types is of considerable help in matching the requisite pairs in more complex expressions. For instance, the expression given in meaning **4** is easier to follow if different brackets are used like this

$$(a + b) \times \{(5a - b) \div [2(7a + 9)]\}$$

The vinculum is still in use today but goes unnoticed. After the root sign √ was introduced (early 16th century) mathematicians would write things like $\surd\ \overline{x + y}$ and it is easy to see what happened next.

In higher mathematics, the vinculum is used to indicate the complex conjugate.

The line used as a fraction divider is not a vinculum, but it is not stretching things unreasonably to consider it that way in the BODMAS context of meaning **4**.

Operator Precedence

The importance of knowing about these rules has changed over time.

Once, after school and examination needs, it was only required by engineers and scientists who used formulas in their work. But the increasing use of computers has meant more people now need to know about, understand and apply that knowledge.

For example, almost anyone using a spreadsheet for some serious purpose could find, among the instructions, a table like this:

Priority	*Operation*	*Symbol*
1	Exponentiation (Indices)	^
2	Multiplication and Division	* /
3	Integer Division	\
4	Addition and Subtraction	+ –

In any single line or statement, operations will be treated with the priority shown. Where operations have the same priority, they will be dealt with in the order in which they occur, in reading the line from left to right. If there might be any ambiguity then brackets should be used. They overide all other priorities.

Conclusions ?

So, which is it to be? As always – it depends. A very familiar teaching situation! But consider

- As a starter, meaning **1** (using the small 'o') might be adequate, or meaning **2** which reduces to the same thing. This may be all that many will require, and it would not be difficult to modify it with an 'upgrade' later.

- There seems to be little point to using meaning **3**

- For those wishing to use it with the more 'formula-like' presentation (as shown) meaning **4** is suitable.

- In view of the need there could be for many to know, or at least be aware of, operator precedence as mentioned earlier, then meaning **5** would seem to be the most useful. But perhaps as BIDMAS to make it unambiguous and avoid that spurious explanation of 'order' or the convoluted contraction relating to 'of'.

An easy way of checking on comparative usage, but with obvious limitations concerning the chosen sample, is to see how much it is used on the Web.
The results (in August 2002) produced by one search-engine were:

Searching for	BODMAS found	684	documents
	BIDMAS	71	
	BODMAS + BIDMAS	12	

How much might those figures change over the coming years?

For another approach to the problem of assigning degrees of precedence to a formula, go to

www.ex.ac.uk/trol/

and look at

=> A Study Guide to the Formulary

where BODMAS is not invoked at all.

Frank Tapson is the author of the *Oxford Mathematics Study Dictionary* and *A Mathematics Formulary.*