Flow diagrams

The use of flow diagrams is covered in most mathematics courses, generally using a ‘real-life’ context of something like making a telephone-call, or cooking an egg, or similar. These are small enough tasks to make the principles evident, but run the risk of trivialising what can an important tool and thus making it seem irrelevant. In this unit the flow diagrams are meant to be directly of use in the classroom and are intended to help pupils do some mathematics more easily. Some pupils might even benefit from having their own set of flowdiagrams to keep in their notebook. It is assumed that a calculator will be used, and it most useful if it has a π key, otherwise a value will have to be given (3.14 will do).

The first four flow diagrams deal with the **SQUARE**, **CIRCLE**, **CUBE** and **SPHERE**.

What these shapes have in common is that, given only one measure (edge-length, area etc.) of that shape then all the others can be found. The degree of difficulty in every case depends upon what is given and what has to be found. For example, in the case of a square, asked to find its perimeter when given its edge-length could be considered an ‘easy’ problem for most. However, if the length of the diagonal is the given measure, then determining the others will be beyond the capabilities of many. But once the use of a suitable flow diagram is invoked then none of the problems are at all difficult. In fact, the biggest problem then is the correct use of the calculator - which is dealt with on a later page and is most important if some errors are to be avoided. See the page headed ‘Calculators and Flow Diagrams’.

Some surprising relationships are sometimes revealed in flow diagrams which are not always apparent in the conventional mathematical methods used in solving problems. Here for instance in the the flow diagram for the square it is effectively saying that area = $2 \times (\text{diagonal})^2$. It is not difficult to demonstrate the truth of this, either geometrically or algebraically, once it had been seen. Following on from this, a similar relationship can then be spotted for the cube. Discoveries concerning such relationships should be encouraged.

A large drawing of the flow diagram for the square is included. This can be used to make an ohp transparency so that the way of working can be demonstrated. Some fun can be had by showing that all squares have the same length of diagonal! Start with an area of 1. Follow through the two instruction boxes to get the length of the diagonal as 1.4142 . . . which it is. Repeat several times with different values for the area and (unless someone knows what this is all about) the diagonal is always 1.4142 . . . For an explanation read ‘Calculators and Flow Diagrams’. Most of the flow diagrams contain this ‘trap’ so it is important to get it sorted out.

The **CONE** and the **PYRAMID** (circular- and square-based respectively) are next. These flow diagrams cover only the simplest relationships between base size, perpendicular height and volume. It merely turns the traditional formulas into flow diagrams with the merit now that ‘reverse’ calculations can be performed without any need for transforming a formula. Measures such as surface area and slant heights are not dealt with here. Pythagoras does not fit into a simple flow diagram. Opportunity could be taken to show how these are in one sense the ‘same’ formula. For any 3-dimensional shape of this kind, having a plane base (whatever its shape) with all the faces coming from that base to a common vertex, its volume is given by the area of that base divided by 3, multiplied by the perpendicular height of that vertex above the base. Note it is the perpendicular height which matters, the vertex does not to have to be located directly over the centre of the base.

The next shape dealt with is the **CYLINDER.** This requires *two* measures to be known, out of five (which means ten possible starting pairs), but some cannot be resolved within the limits of a simple flow diagram so there are some calculations which are not offered here. For example, given the total surface area and volume as starting measures, it requires the solution of a cubic equation to find the others. And this is beyond normal school mathematics, never mind simple flow diagrams. It does need to be made clear that the first requirement is to find where the two pieces of given data occur. One of them being used in the starting-box and the other in a subsequent instruction-box. Also, in some cases, a value not expressly asked for in a question may have to be found in order to use it elsewhere in the flow diagram. For example, given the total surface area and the radius, and asked to find the curved surface area (which uses the right-hand diagram of the two) then it would first be necessary to find the height (using the left-hand diagram).

continued . . .

The **SECTOR of a Circle** also requires *two* measures to be known, but nearly all of the possible calculations are on offer, though some thought is needed in a few cases.

The **OBLONG** requires *two* measures to be known, but much more thought needs to be given to teasing out just how to use this one effectively, especially where some things have to be worked out and then used elsewhere. A new idea here is when given the diagonal and one edge it requires two values to be calculated in parallel, and then multiplied together. A few things that cannot be done within the restrictions of a simple flow diagram are, starting with any two of Area, Perimeter and Diagonal, the edge-lengths cannot be found.

The **CUBE ROOT** flow diagram is something different. It was originally produced to help those who had only a basic calculator. The increasing availability of scientific calculators might suggest it is now redundant. However, it still has a purpose in showing the use of a decision-box and a loop.
For more-able pupils some discussion on why it works might be useful. It is an example of an iteration, repeating a process with ever increasing accuracy. It is based on the fact that we are seeking a solution to $x^3 = N$ which we can re-write as $x = N \div x^2$ and then using this latter equation by putting in a first approximation for x, evaluating $N \div x^2$ to give a closer value for x and then using that, and so on.

PRACTICE EXERCISES in the form of tables needing to be completed provide plenty of work to gain familiarity with these flow diagrams. They cover six out of the nine flow diagrams given. Impossible cases are blanked out. Answers are provided, to be used in any way considered appropriate. They could be given out at the same time as the exercises for a first practice. Answers are also included for the 'impossible' cases for the benefit of those who might have been able to do them by some other means (trial and error?).

Additional Notes

While flow diagrams should be self-contained and self-explanatory, an introduction is bound to be needed. The style and depth of that introduction must depend upon the class and will vary widely.

There are two sorts of 'boxes'. The rectangles are 'action-boxes' containing an instruction to do something. The others are 'dimension-boxes' containing the name of one of the measures of the shape being dealt with. The instruction in an action-box always has to be applied to the worked-out value at that point. For a detailed explanation of what that means see 'Calculators and Flow Diagrams'. It must be understood that in all the boxes the wording is economical. For example:

'**Radius**' *means* 'the length of the radius'

'**Square it**' *means* 'multiply the number now showing on the calculator display by itself'.
This can usually be done by pressing [×] [=]

$\sqrt{\ }$ *means* 'take the square root of the number now showing on the calculator display.
An understanding of what this entails in a flow diagram is essential.

Also

A **space diagonal** is a straight line in a 3-dimensional shape joining two vertices which are not on the same face.

Units tend to be ignored, but some mention should be made of the fact that they must be consistent; to have a radius in millimetres and a height in centimetres is not a good idea. Also that the units of the final answer will depend upon the input; if the length of a diagonal is given in metres then the area will be in square metres. Both of these points are the same whatever method is being used to solve the problem of course, but seems to be much more easily overlooked when working with a flow diagram.

Calculators and Flow Diagrams

Leaving aside more esoteric types (such as graphics etc.) there are broadly two types of calculator used to do arithmetic. One is usually described as 'Basic', the other as 'Scientific'. They can usually be distinguished quite easily at a glance by the complexity of their respective keyboards. However, a much more important distinction for the user is in the way they operate, and everyone ought to be aware of that difference. The Scientific Calculator understands that, in mathematics, there are orders of precendence when using a combination of operations like, for instance, multiplication has to be done before addition. The Basic Calculator knows nothing of this and works things out in the order in which they are given. To see this use the 234-Test.

The 234-Test

Consider the sum $2 + 3 \times 4$

Mathematically, since multiplication has precedence over addition, the 3×4 must be done first to give 12, and then the 2 is added to make 14

However, simple reading from left to right yields the working: $2 + 3$ gives 5, which is multiplied by 4 to make 20

Two different answers to the same statement - an unacceptable anomaly.

Try it on a calculator, with these key presses:

[2] [+] [3] [×] [4] [=]

On a Scientific Calculator the answer will be 14

On a Basic Calculator the answer will (usually) be 20

Since this difference exists, and must be dealt with in some way, it is worth giving a particular name to one of them. The 'correct' way (using the mathematical rules of operator precedence) is simply 'arithmetic'. The other way (working from left to right) will here be referred as '**flow-arithmetic**'.

Can the Scientific Calculator be made to do flow-arithmetic? Quite easily. In the above case, pressing:

[2] [+] [3] [=] [×] [4] [=] gives the answer 20

The use of the extra [=] has forced it to work out $2 + 3$ *before* going on to $\times 4$

An operator which complicates matters (on any calculator) is $\sqrt{}$

Consider $\sqrt{5+4}$ which can be seen as $\sqrt{9}$ which is 3

Even with flow-arithmetic, since it is the square-root of the number(s) inside the sign which has to be found, it must be realised that [$\sqrt{}$] has to come at the end, rather than the beginning, so it is done on the calculator as:

[5] [+] [4] [$\sqrt{}$] [=] which gives the answer 7. What has gone wrong?

Studying the calculator display as the keys are pressed shows

Key press:	[5]	[+]	[4]	[$\sqrt{}$]	[=]
Display:	5	5	4	2	7

This demonstrates that the key [$\sqrt{}$] operates only on the number in the display at the time, regardless of anything that has gone before.

Again, the insertion of an extra [=] will force the correct working. Press:

[5] [+] [4] [=] [$\sqrt{}$] and the answer is 3

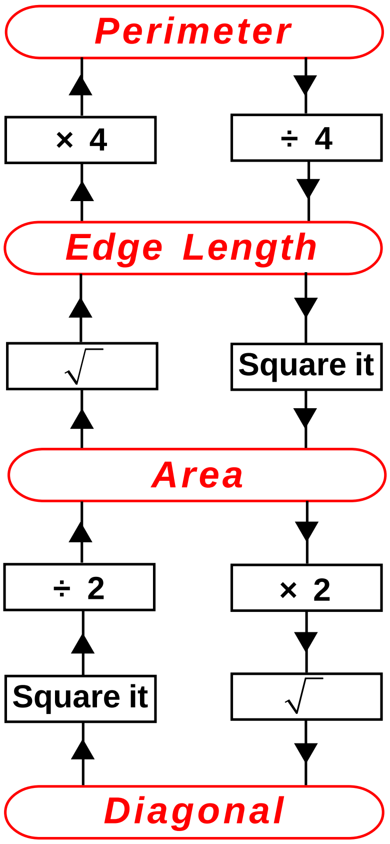
Notice there is no need for a final [=]. In fact, on some calculators it can be disastrous! Try it.

An (almost) fool-proof rule in flow-arithmetic is

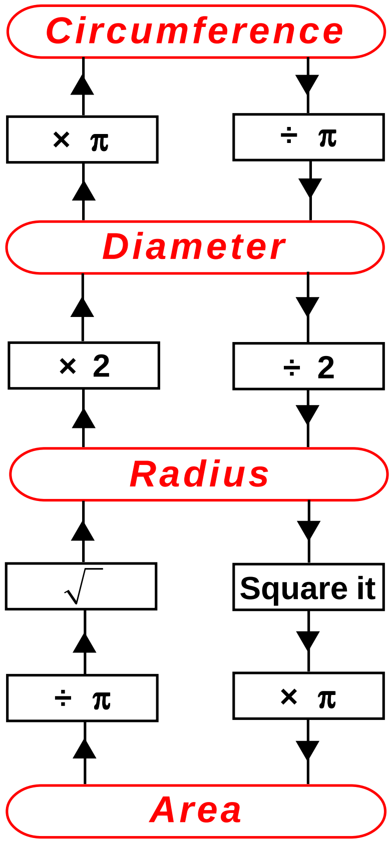
Always press [=] after a (complete) **number has been entered.**

Caution. *In spite of all the definite statements made above, they are only generalisations to provide guidance. Calculators do vary and need to be 'tested' to establish their way of working.*

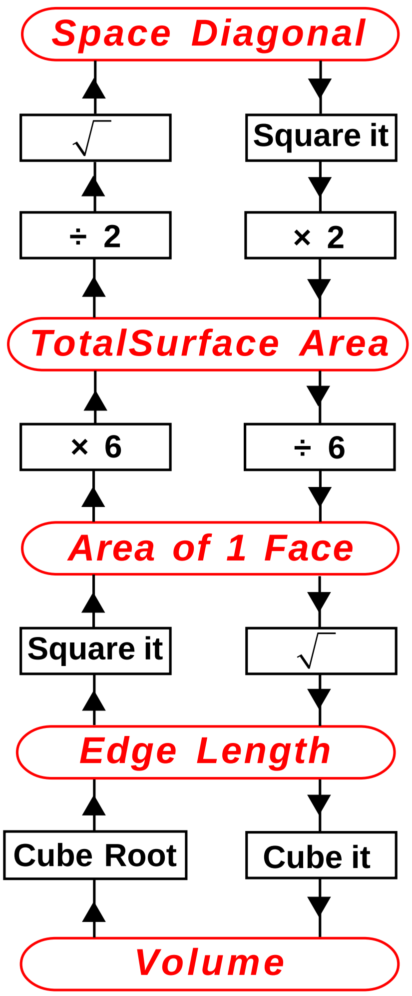
SQUARE



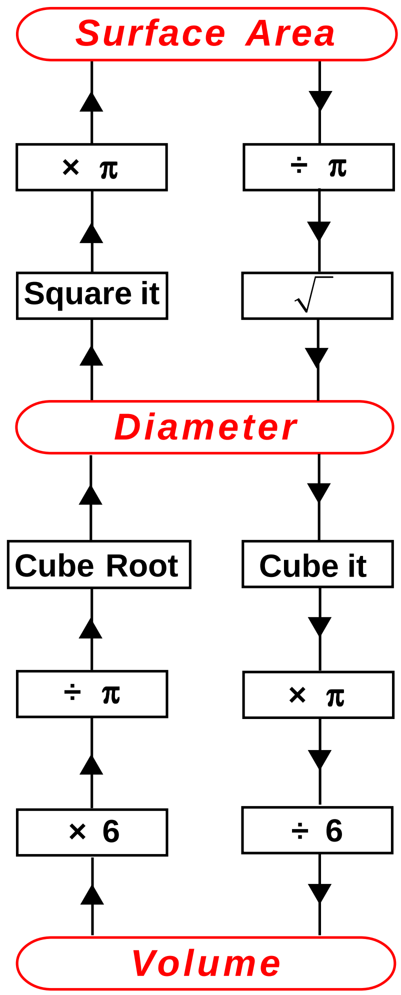
CIRCLE



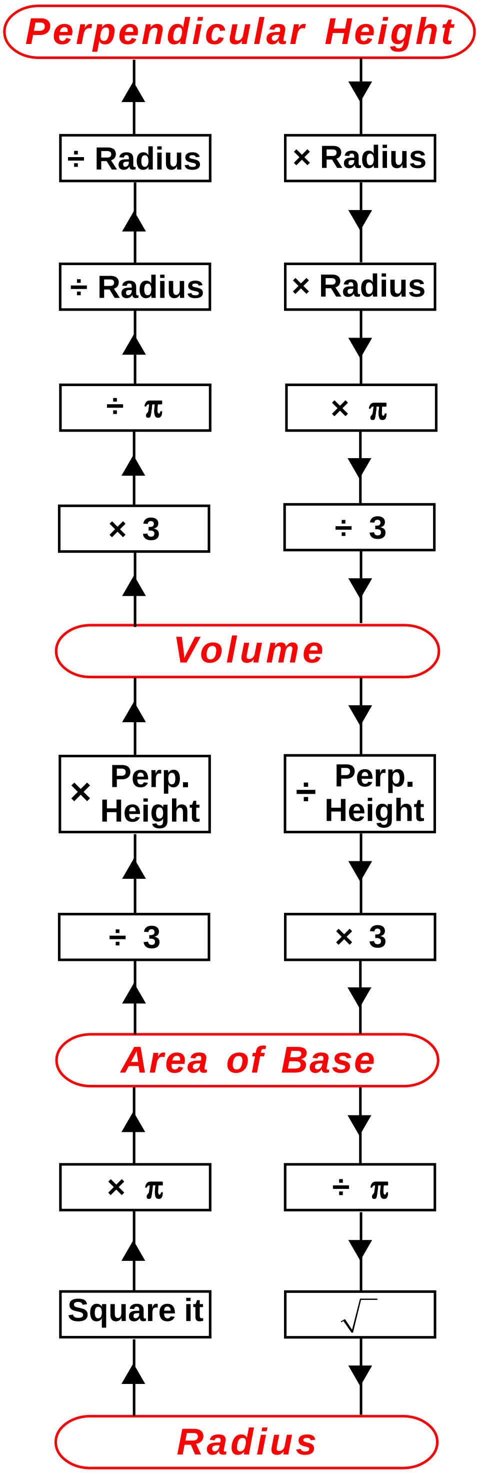
CUBE



SPHERE

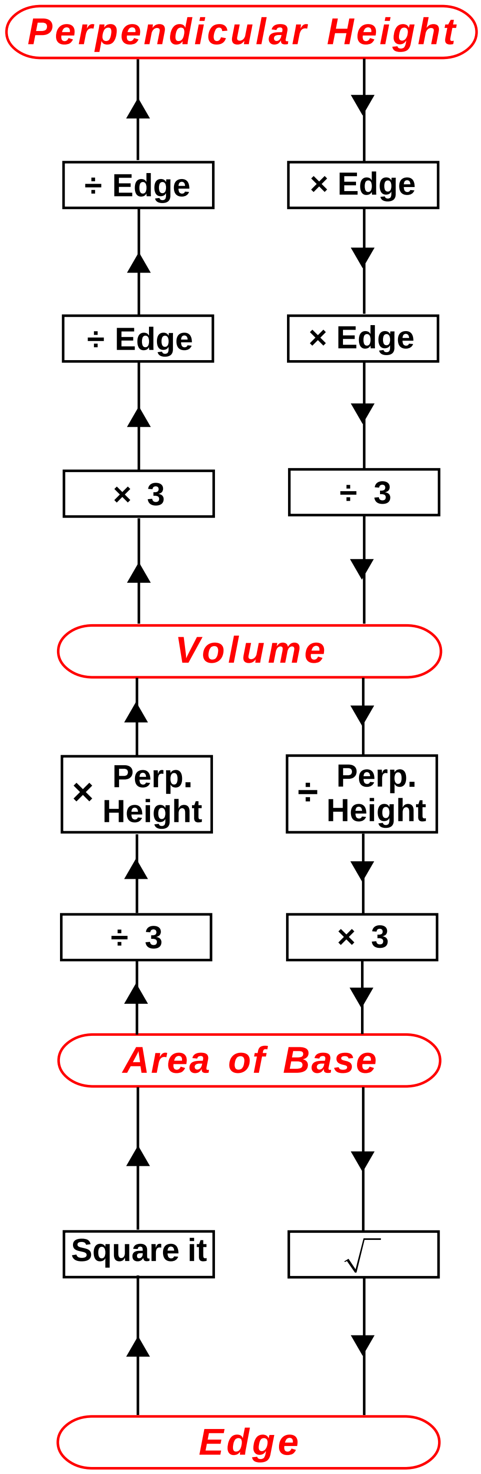


CONE
Circular - based



Radius = Radius of Base Circle

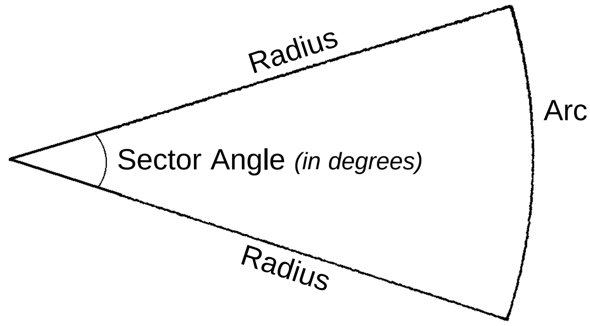
PYRAMID
Square - based



Edge = Edge of Base Square

Perp. Height = Perpendicular Height of Shape

SECTOR of a Circle



Sector Angle

÷ Area	× Area
÷ π	× π
× 90	÷ 90
Square it	$\sqrt{\ }$

Arc Length

÷ 180	× 180
× π	÷ π
× Radius	÷ Radius

Sector Angle

÷ Radius	× Radius
÷ Radius	× Radius
÷ π	× π
× 360	÷ 360

Area of Sector

÷ 2	× 2
× Radius	÷ Radius

Arc Length

Arc Length

÷ 180	× 180
× π	÷ π
× Sector Angle	÷ Sector Angle

Radius

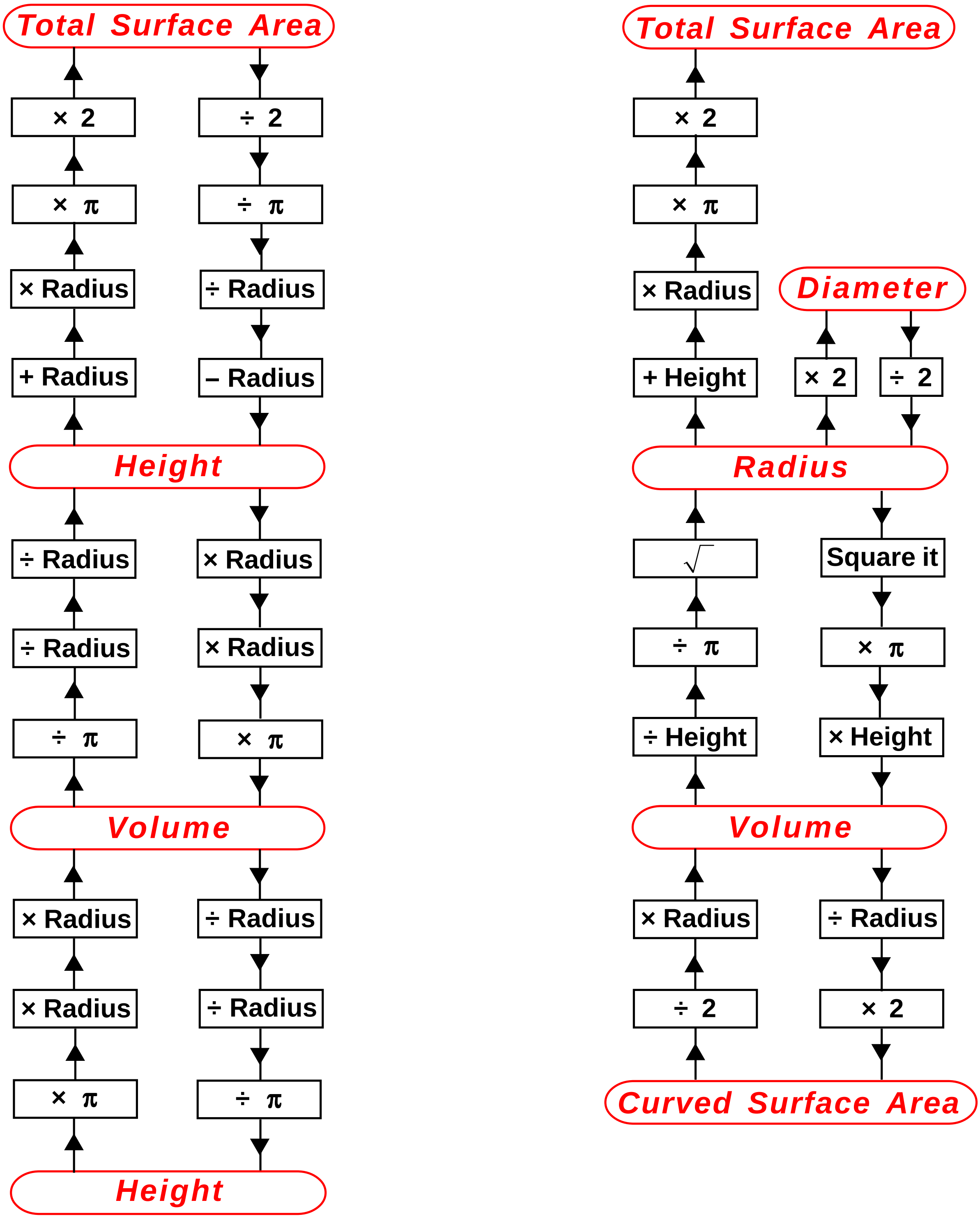
$\sqrt{\ }$	Square it
× 360	÷ 360
÷ π	× π
÷ Sector Angle	× Sector Angle

Area of Sector

÷ 2	× 2
× Arc Length	÷ Arc Length

Radius

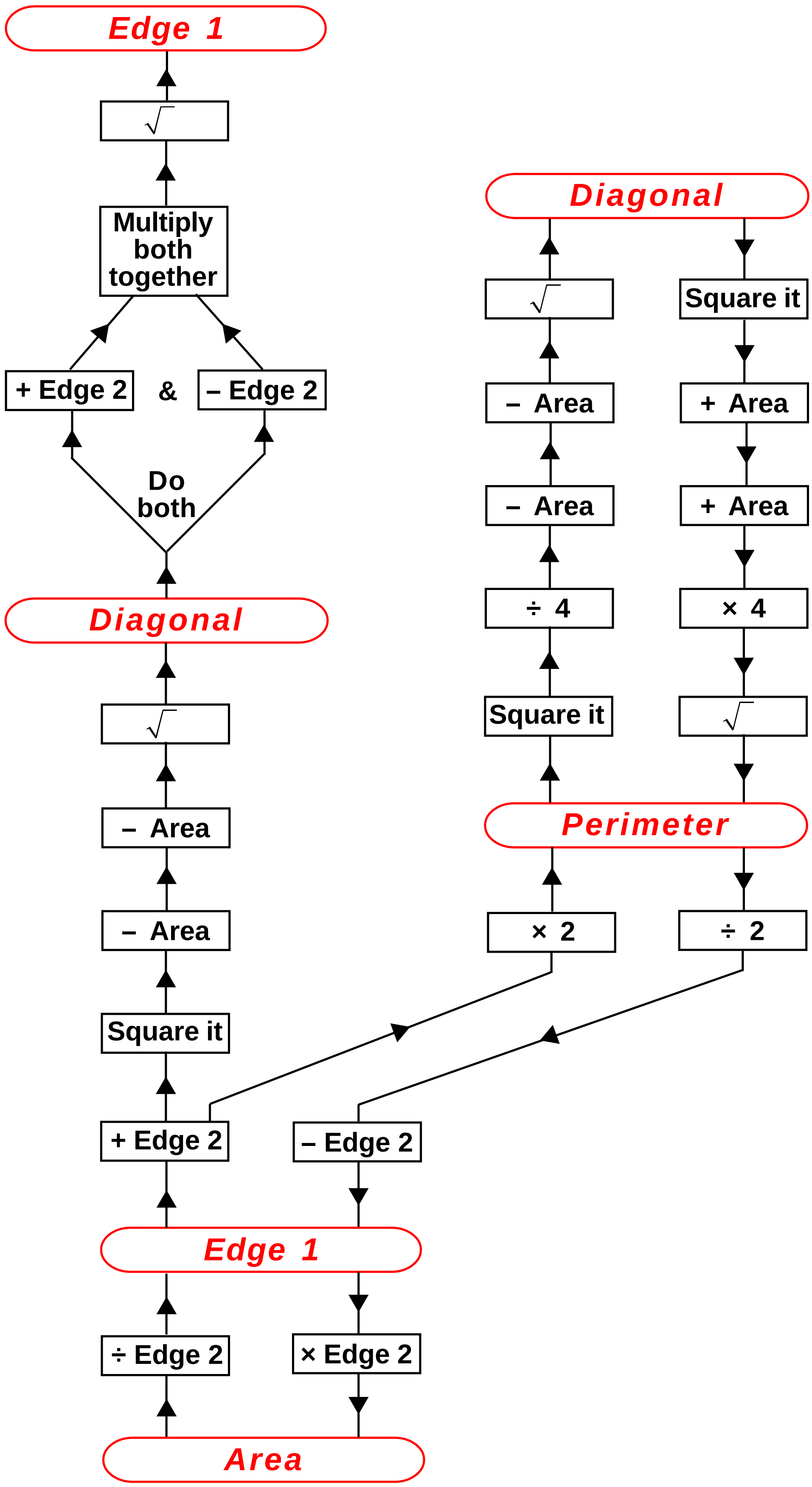
CYLINDER



Note: ***Height*** might be called ***Length***

OBLONG

An oblong is a rectangle which has two **different** lengths of edge. The edges are referred to here as Edge 1 and Edge 2, but for calculating purposes, their names (or values) are interchangeable.



CUBE ROOT

The flow diagram on the right can be used to find the cube root of any number very quickly.

It works by making an estimate of the cube root, and then using that estimate to get a more accurate estimate, and then using that estimate to get an even more accurate estimate and so on.

N = Number whose cube root is to be found.

To decide when the answer is accurate enough:

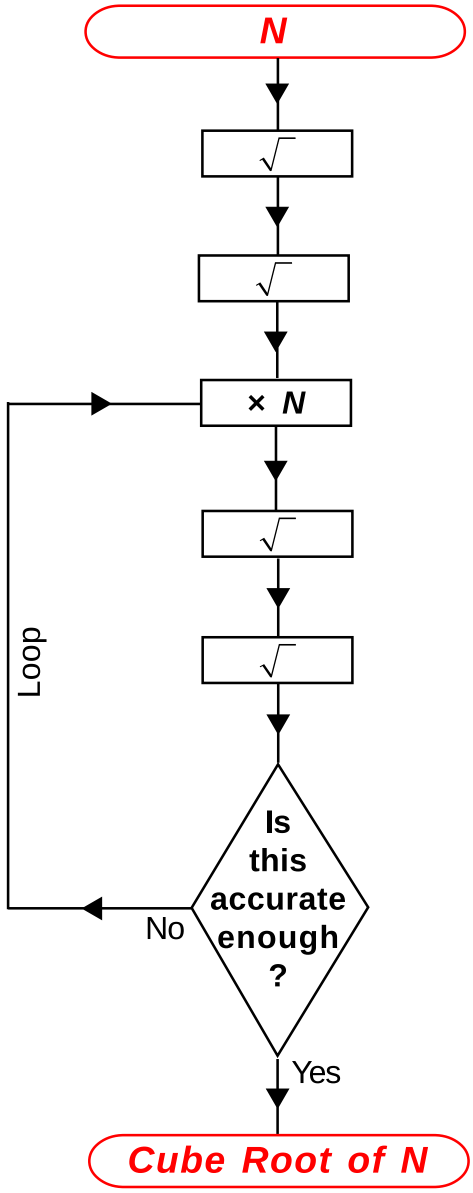
First decide, to how many decimal places the answer is needed and add one (to allow for rounding).

Then, each time the question-box is reached, make a note of the answer before going around the 'loop'.

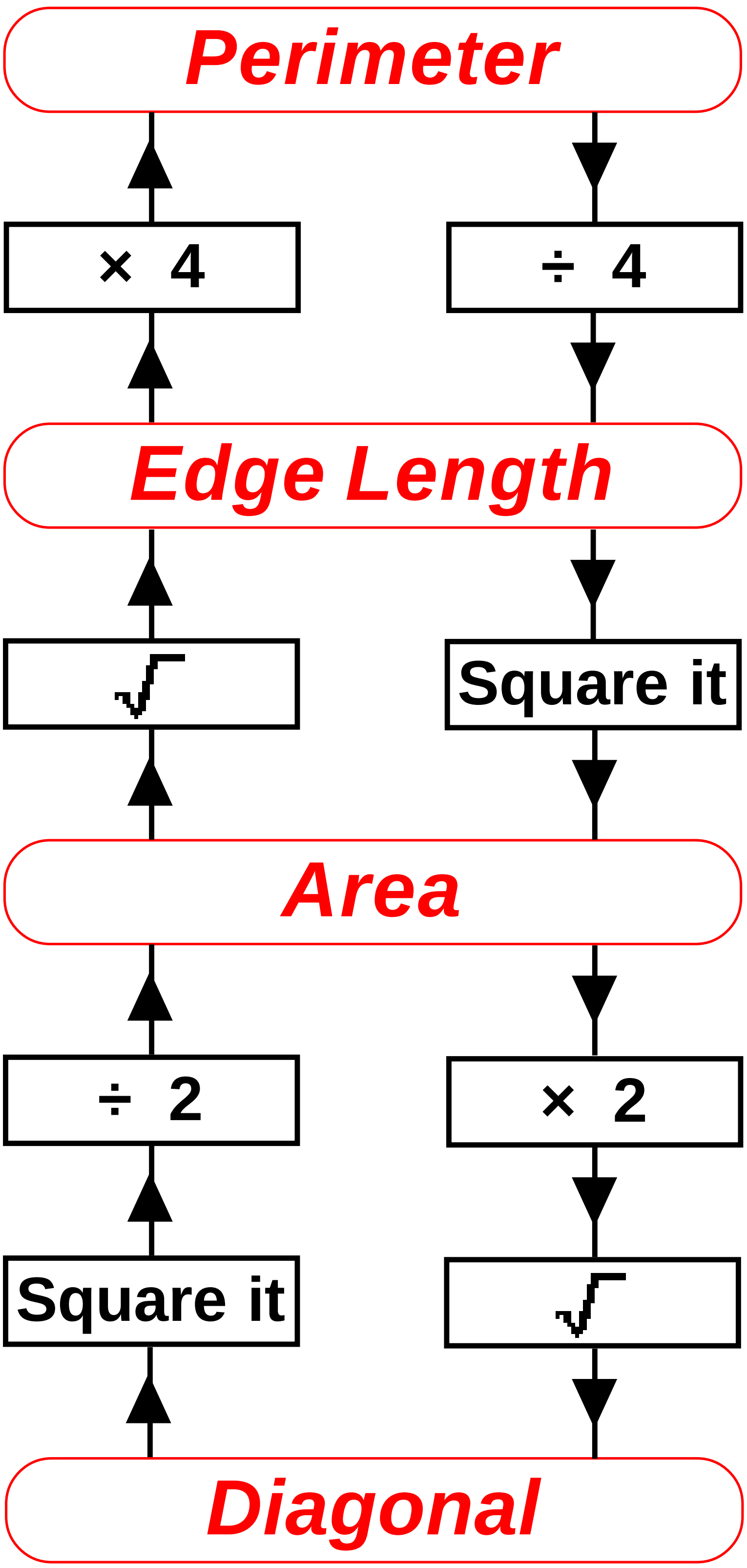
When an answer repeats itself to the number of decimal places decided upon, then it is accurate enough.

As an example, searching for the cube root of 40 produces these answers at the question-box stage (different calculators might vary on the last digit) and shows the number of decimal places to which they match:

3.1669685	
3.3548718	
3.4035645	1 d.p.
3.4158477	
3.4189254	
3.4196952	3 d.p.
3.4198877	
3.4199359	4 d.p.
3.4199479	
3.4199509	5 d.p.
3.4199516	
3.4199518	
3.4199519	7 d.p.
3.4199519	



SQUARE


Perimeter
× 4
÷ 4
Edge Length
√
Square it
Area
÷ 2
× 2
Square it
√
Diagonal

Flow Diagram Practice Exercises ~ 1

Complete these tables of information for the shapes named. Give all answers to 3 decimal places.

Square

	Edge Length	Area	Perimeter	Diagonal
1	5.6 cm			
2		19.0 cm^2		
3			27 cm	
4				12.5 m
5		147.3 mm^2		
6				78 cm

Circle

	Radius	Diameter	Circumference	Area
7	2.6 cm			
8		7.8 cm		
9			31.4 m	
10				100.0 cm^2
11			457 mm	
12				9.56 m^2

Sector of a Circle

	Radius	Sector Angle	Area	Arc Length
13	5 cm	40^{o}		
14	7.5 m			3.2 m
15		74^{o}		16 mm
16	9.6 cm		8.70 cm^2	
17		115^{o}	26.3 m^2	
18			51.1 cm^2	9.4 cm

Flow Diagram Practice Exercises ~ 2

Complete these tables of information for the shapes named. Give all answers to 3 decimal places.

Cube

	Edge Length	Area of 1 Face	Total Surface Area	Space Diagonal
19	2.8 cm			
20		17.0 cm ^{2}		
21			2143.0 mm ^{2}	
22				9.2 m
23		156.4 mm ^{2}		
24				78 cm

Sphere

	Diameter	Surface Area	Volume
25	2.65 cm		
26		43.7 cm ^{2}	
27			54.6 mm ^{3}

Oblong

	Edge 1	Edge 2	Area	Perimeter	Diagonal
28	3 cm	4 cm			
29	3 cm	5 cm			
30	4.5 mm		43.0 mm ^{2}		
31	7.86 m			28.4 m	
32	[redacted]	[redacted]	57.0 cm ^{2}		12.7 cm
33		12 cm		78 cm	
34		34.6 mm	210.4 mm ^{2}		
35		3.2 m			6.8 m
36	[redacted]	[redacted]	71.2 cm ^{2}		13.4 cm

Flow Diagram Practice Exercises ~ 1 ~ ANSWERS

All answers given to 3 decimal places.

Square

	Edge Length	Area	Perimeter	Diagonal
1	5.6 cm	**31.360 cm ^{2}**	**22.400 cm**	**7.920 cm**
2	**4.359 cm**	19.0 cm ^{2}	**17.436 cm**	**6.164 cm**
3	**6.750 cm**	**45.563 cm ^{2}**	27 cm	**9.546 cm**
4	**8.839 m**	**78.125 m ^{2}**	**35.355 m**	12.5 m
5	**12.137 mm**	147.3 mm ^{2}	**48.547 mm**	**17.164 mm**
6	**55.154 cm ^{2}**	**3042.000 cm ^{2}**	**220.617 cm**	78 cm

Circle

	Radius	Diameter	Circumference	Area
7	2.6 cm	**5.200 cm**	**16.336 cm**	**21.237 cm ^{2}**
8	**3.900 cm**	7.8 cm	**24.504 cm**	**47.784 cm ^{2}**
9	**4.997 cm**	**9.995 cm**	31.4 m	**78.460 cm ^{2}**
10	**5.642 cm**	**11.284 cm**	**35.449 cm**	100.0 cm ^{2}
11	**72.734 mm**	**145.468 mm**	457 mm	**16619.675 mm ^{2}**
12	**1.744 m**	**3.489 m**	**10.961 m**	9.56 m ^{2}

Sector of a Circle

	Radius	Sector Angle	Area	Arc Length
13	5 cm	40 ^{o}	**8.727 cm ^{2}**	**3.491 cm**
14	7.5 m	**24.446 ^{o}**	**12.000 cm ^{2}**	3.2 m
15	**12.388 mm**	74 ^{o}	**99.106 m ^{2}**	16 mm
16	9.6 cm	**10.818 ^{o}**	8.70 cm ^{2}	**1.813 cm**
17	**5.119 m**	115 ^{o}	26.3 m ^{2}	**32.492 m**
18	**10.872 cm**	**49.537 ^{o}**	51.1 cm ^{2}	9.4 cm

Flow Diagram Practice Exercises ~ 2 ~ ANSWERS

All answers given to 3 decimal places.

Cube

	Edge Length	Area of 1 Face	Total Surface Area	Space Diagonal
19	2.8 cm	**7.840 cm ^{2}**	**47.040 cm ^{2}**	**4.850 cm**
20	**4.123 cm**	17.0 cm ^{2}	**102.000 cm ^{2}**	**7.141 cm**
21	**18.899 mm**	**357.167 mm ^{2}**	2143.0 mm ^{2}	**32.734 cm**
22	**5.312 m**	**28.213 m ^{2}**	**169.280 m ^{2}**	9.2 m
23	**12.506 mm**	156.4 mm ^{2}	**938.400 mm ^{2}**	**21.661 mm**
24	**45.033 cm**	**2028.000 cm ^{2}**	**12168.000 cm ^{2}**	78 cm

Sphere

	Diameter	Surface Area	Volume
25	2.65 cm	**22.062 cm ^{2}**	**9.744 cm ^{3}**
26	**3.730 cm**	43.7 cm ^{2}	**27.164 cm ^{3}**
27	**4.707 mm**	**69.601 mm ^{2}**	54.6 mm ^{3}

Oblong

	Edge 1	Edge 2	Area	Perimeter	Diagonal
28	3 cm	4 cm	**12.000 cm ^{2}**	**14.000 cm**	**5.000 cm**
29	3 cm	5 cm	**15.000 cm ^{2}**	**16.000 cm**	**5.831 cm**
30	4.5 mm	**9.556 mm**	43.0 mm ^{2}	**28.111 mm**	**10.562 mm**
31	7.86 m	**6.340 m**	**49.832 m ^{2}**	28.4 m	**10.098 m**
32	**4.858 cm**	**11.734 cm**	57.0 cm ^{2}	**33.184 cm**	12.7 cm
33	**27.000 cm**	12 cm	**324.000 cm ^{2}**	78 cm	**29.547 cm**
34	**6.081 cm**	34.6 mm	210.4 mm ^{2}	**81.382 cm**	**35.130 cm**
35	**6.000 m**	3.2 m	**19.200 m ^{2}**	**18.400 m**	6.8 m
36	**5.924 cm**	**12.020 cm**	71.2 cm ^{2}	**35.866 cm ^{2}**	13.4 cm