Mensuration

This a mixed selection of worksheets on a standard mathematical topic. A glance at each will be sufficient to determine its purpose and usefulness in any given situation. These notes are intended only to provide some additional background in a few of the cases, together with some suggestions. These sheets are probably not suitable for beginners, nor to serve as introductory exercises, but would probably best be suited for use as a major revision. Many of them are of a ‘practical’ nature. The others all require an ability to manipulate the appropriate formulas.

The shapes covered are

2-dimensional	**square**	**circle**	**sector**
3-dimensional	**cube**	**cylinder**	**sphere**

Further work on circles can be found under Circles and also the Dartboard.
For more complex examples generally look at Graduated Problems.

The sheets rarely make any stipulation about the degree of accuracy to be used, but suitable instructions should be given. What value is to be used for π? How accurately should the answer be stated - to 3 significant figures, 2 decimal places, or as seems appropriate? What accuracy is expected in the measurements where they are necessary? How are the calculations to be done - by formula or by flow-diagram (where that is possible)?

Where sheets (or half-sheets) exists in an A and a B version they could be used by giving one as a practice and the other as a test. Or, they could be used as a more secure test by giving out both at the same time, only making sure that adjacent users do not have the same version.

Cubes and **spheres** have been put together on the same sheet since they both involve cubing a number and also finding a cube root. The term ‘space-diagonal’ is used here for the diagonal which goes between two opposite vetices not in the same face. You might consider the advantages of using the diameter of a sphere for its mensuration. The volume $\frac{\pi d^3}{6}$ is much more compact and elegant than $\frac{4\pi r^3}{3}$ and has the added advantage of showing at a glance that a sphere is approximately one-half the volume of its circumscribing cube. Its surface area is then simply πd^2. Also, a practical detail, the diameter of a sphere is much more easily measured that its radius.

Practical Cylindricals

This is a very good practical lesson on cylinders based on a miscellaneous collection of old (but clean!) tin-cans, but the preparatory workload for this is rather large so that it is probably only worth doing if it is to be used several times and/or others can be persuaded to share in the preparation (and usage). Get a variety of tins of various sizes, enough for at least one can between two pupils. Number the cans - a permanent ohp marker pen will serve for this. Issue the cans, together with a copy of the sheet bearing the title ‘Practical Cylindricals’. Again, the issue of accuracy (and π) must be addressed. The marking is relatively easy - but only once it has been set up!
In connection with Qn 7 on the sheet, this could lead to an investigation on other possible layouts for cutting the parts of a can from a single rectangle and the different percentages of waste.

Cylinders is probably the hardest sheet in this unit. It makes considerable demands on the ability to carry out algebraic manipulation - at least if analytical solutions are being attempted. Solutions can be sought by trial and error methods, but that is very tedious. In some cases (but certainly not all) a suitable flow-diagram can be found - which makes it relatively easy. The last two questions on this sheet are particularly demanding.

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Pay particular attention to the units used (in the question) when giving the final answer.

1. A square has an edge of length of 5 cm. What is its perimeter?
2. A square has a perimeter of 40 cm. What is its edge-length?
3. The edge of a square measures 6 metres. Calculate its area.
4. What is the edge-length of a square having an area of 64 mm^2?
5. Find the length of the diagonal of a square whose edge-length is 7 km.
6. The diagonal of a square measures 12 cm. What is the length of one edge?
7. Calculate the length of a diagonal of a square having an area of 45 m^2
8. A square has an area of 20 mm^2. What is its perimeter?
9. Find the area of a square whose perimeter is 80 mm.
10. A square has a diagonal measuring 10.7 cm. Calculate the area of the square.
11. For a square having an area of 730 mm^2, find the length of its diagonal.
12. What must be the length of the perimeter of a square whose diagonal measures 7.82 metres?

Drawing squares

To help in setting out the drawings required in answer to these questions, it is best to use the sheet headed **Squares (Layout Sheet)**. On that sheet the top left-hand corner of most of the squares that have to be drawn are marked with the same letter as used to identify the question here. As a check, no edges of any squares should cross or coincide with any other.

A Draw a square with an edge-length of 3 cm

B Draw a square with an area of 16 cm^2

C Draw a square with a perimeter of 25 cm

D Draw a square having an area of 12.3 cm^2

E Draw a square whose diagonal measures 6 cm

F Draw a square which has an area of 49 cm^2
Inside **F** draw a square which has only half that area. Label it **G**

H Draw a square whose area is 1600 mm^2

J Draw a square which has a perimeter of 34 cm
Inside **J** draw a square having an area 34 cm^2. Label it **K**

L Draw a square with an edge-length of 80 mm
Inside **L** draw a square whose diagonal is 80 mm. Label it **M**
Inside **M** draw a square having an area 800 mm^2. Label it **N**

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Squares (Layout Sheet)

A

B

C

D

F

H

E

J

L

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A Complete the following table of data for 12 **circles**

	Circumference	Diameter	Radius	Area
1			*3*	
2		*12*		
3		*7.3*		
4		*8.29*		
5	*24.6*			
6	*16*			
7		*31.5*		
8				*24.8*
9				*76*
10	*124*			
11				*263*
12		*357*		

B Complete the following table of data for 12 **circles**

	Circumference	Diameter	Radius	Area
1			*1*	
2		*8*		
3		*6.7*		
4		*9.23*		
5	*21.3*			
6	*19*			
7		*37.9*		
8				*27.7*
9				*59*
10	*143*			
11				*378*
12		*245*		

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All the questions on this sheet concern the drawings on the separate sheet headed
Circles & Sectors (Drawings).
All linear measurements needed to solve the problems given here should be taken from the drawings on that sheet, working to the nearest millimetre.
For angles, use the size stated on the drawing.
Pay particular attention to the units asked for (in the question) when giving the final answer.

Calculate

1. for drawing **A**
   (a) the shaded area (in cm^2)
   (b) the circumference (in mm).

2. for drawing **B**
   (a) the area (in cm^2) of the larger circle
   (b) the area (in cm^2) of the smaller (unshaded) circle
   (c) the shaded area (in cm^2).

3. for drawing **C**
   (a) the total unshaded area (in cm^2)
   (b) the shaded area (in cm^2)
   (c) the shaded area (in mm^2).

4. for drawing **D**
   (a) the arc length (in mm) of the sector
   (b) the total perimeter (in mm) of the sector
   (c) the area (in cm^2) of the sector.

5. for drawing **E**
   (a) the area (in mm^2) of the shaded sector
   (b) the total perimeter (in mm) of the unshaded sector.

6. Drawing **F** has been made by drawing 4 quarter-circles inside a square.
   For the shaded part shown, calculate
   (a) its perimeter (in mm)
   (b) its area (in mm^2).

7. Drawing **G** has been made by drawing 3 semi-circles with all their centres falling on the same straight (dashed) line.
   What is the size of the shaded area (in cm^2)?

8. Drawing **H** is made up of 2 semi-circles and a sector of a circle.
   Calculate
   (a) its area (in cm^2)
   (b) its perimeter (in cm).

9. Drawing **J** is made up of 2 semi-circles and a sector of a circle.
   Calculate
   (a) its area (in cm^2)
   (b) its perimeter (in cm).

10. What is the total of all the shaded areas (in cm^2) on the sheet?

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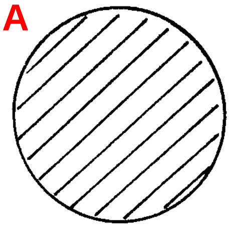
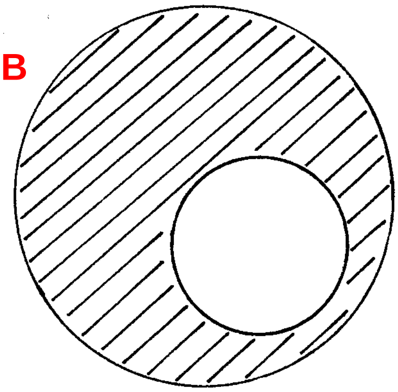
Circles & Sectors
(Drawings)

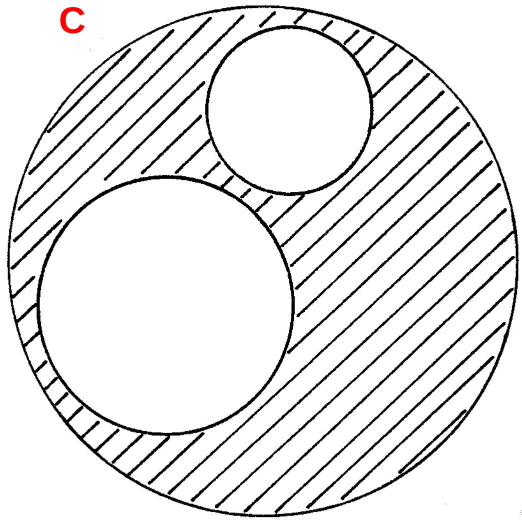
A



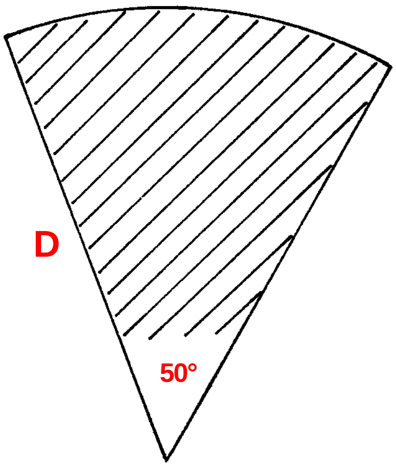
B



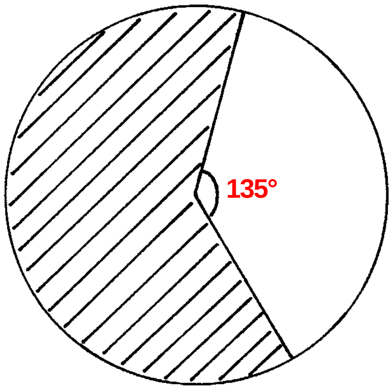
C



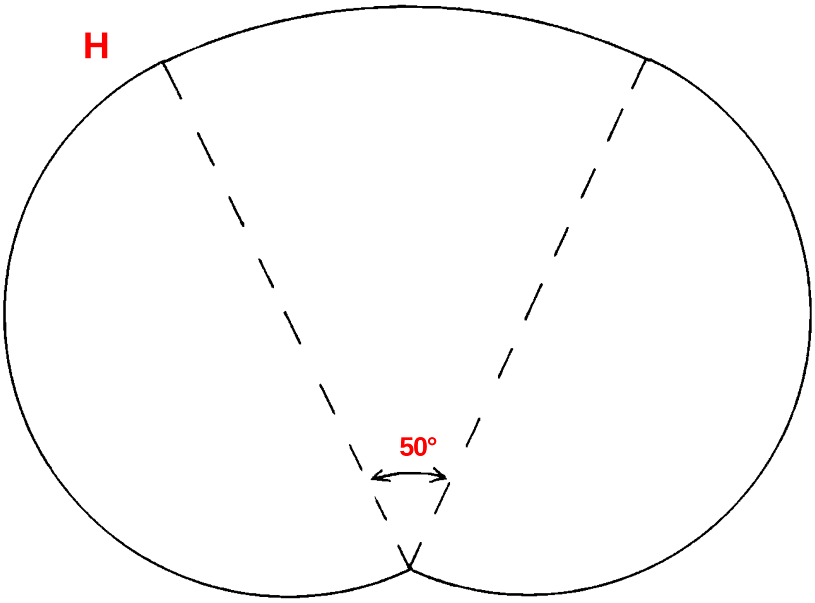
D



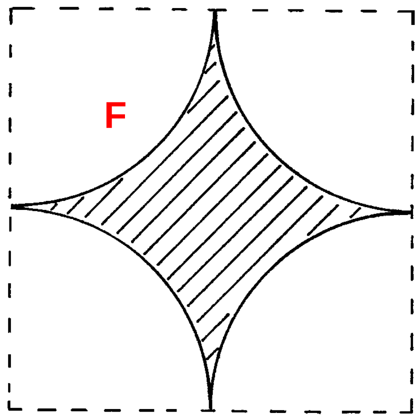
E



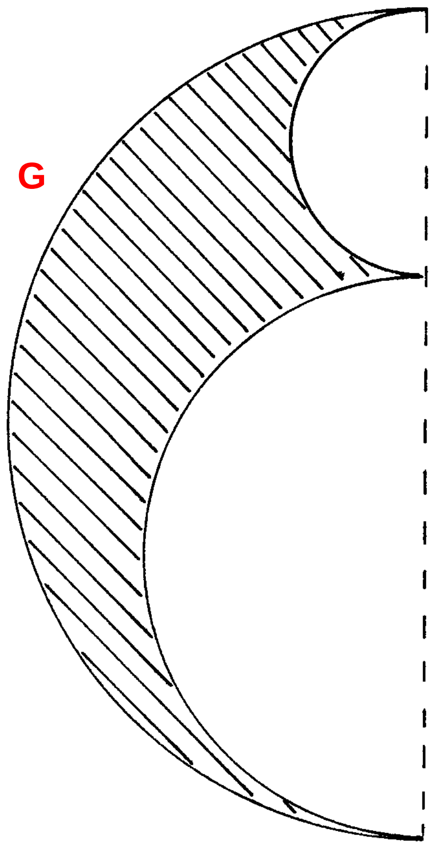
H



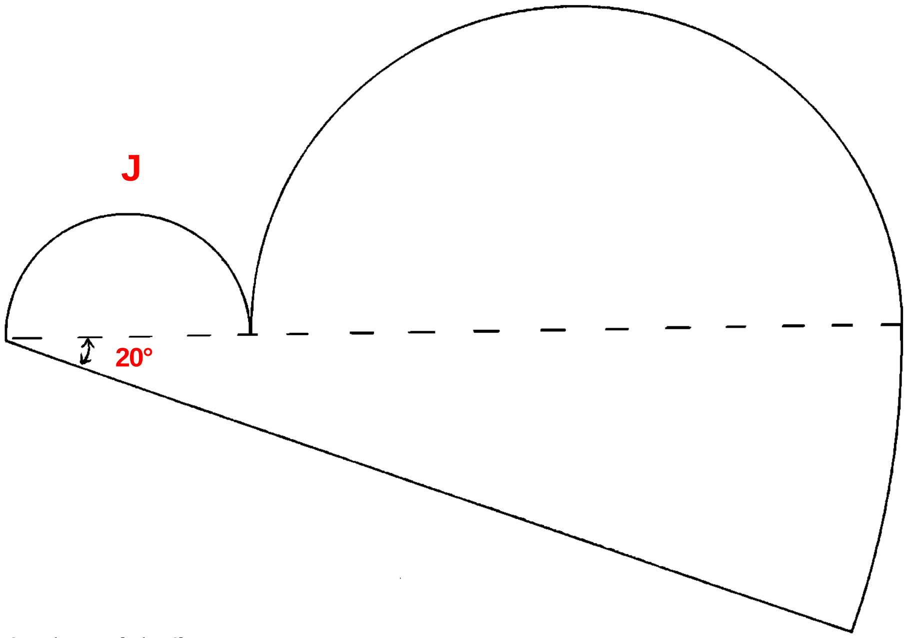
F



G



J



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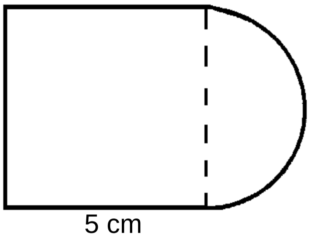
The shapes in drawings 1 to 15 are based on squares, half-squares, circles, semi-circles and quarter-circles. The outline of the complete shape is shown by the solid line; broken lines are there only to show how the overall shape is made up.
The drawings are not to scale.
Calculate the area and perimeter of each of the shapes 1 to 11.

1



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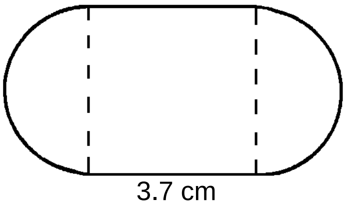
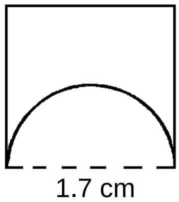
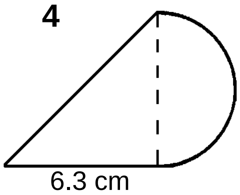


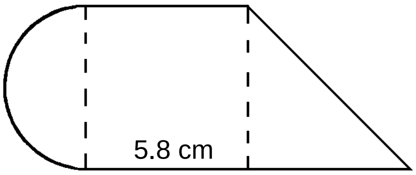
3



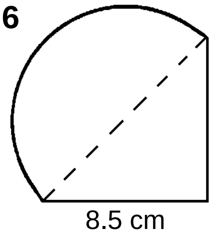
4



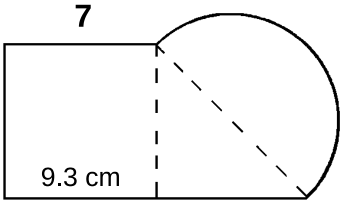
5



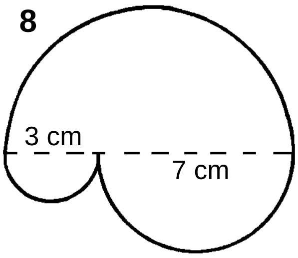
6



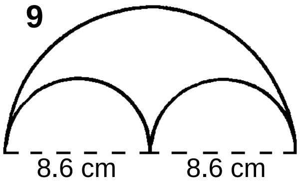
7



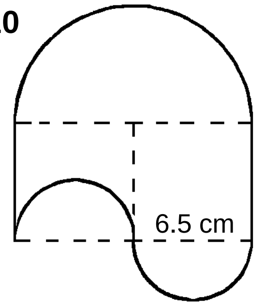
8



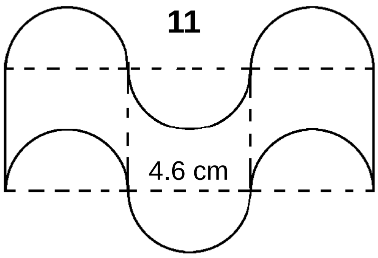
9



10

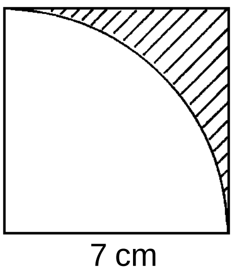


11

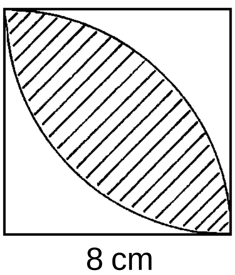


For each of these drawings, 12 to 15, calculate the shaded area.

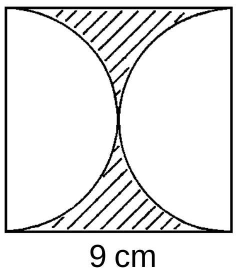
12



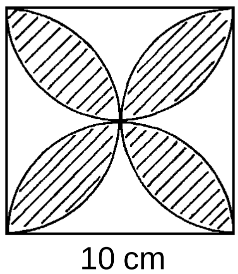
13



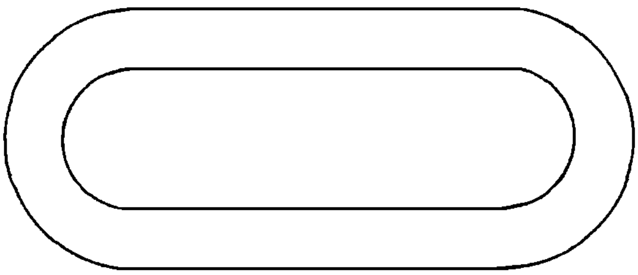
14



15



The drawing on the right, which is not to scale, is of a running track. The ends are semi-circular.
The distance between the two inner parallel staight lines is 60 metres, and they are each 106 metres long. The track is 10 metres wide.
Calculate (a) the distance around the track along its inner edge.
(b) the area of the actual track



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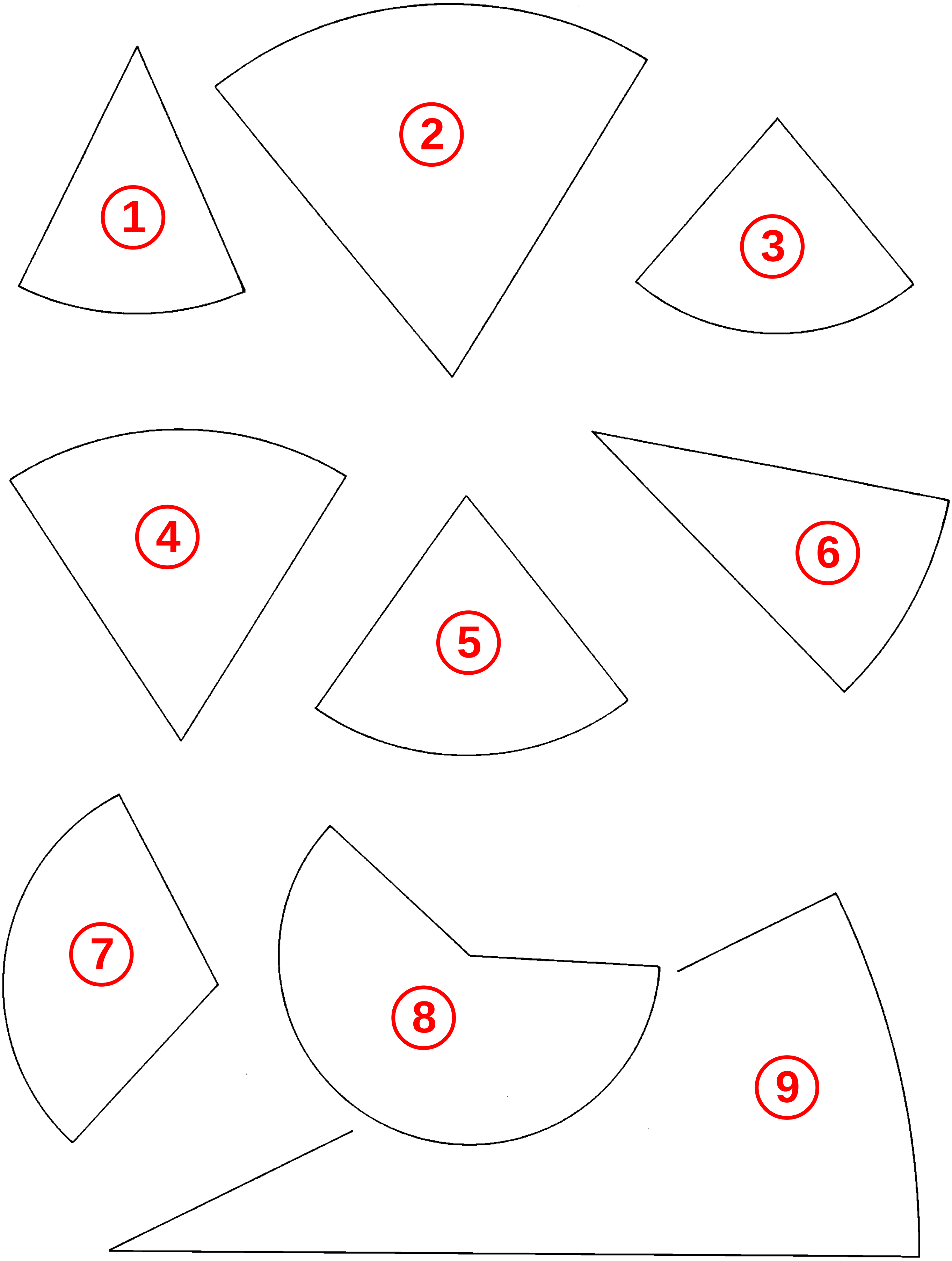
For each of the sectors (1 to 9) below, measure

(a) the sector angle (b) the radius used to draw it

and then, using those measurements, calculate

(c) the arc length (d) the area of the sector



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1. What is the volume of a cube having an edge length of 2.6 cm?

2. A cube has an edge of length 3.4 cm. What is the length of its 'space-diagonal'?

3. The edge-length of a cube is 1.7 cm. Find its total surface area.

4. Calculate the length of edge of a cube which has a total surface area of 86 cm^2.

5. The 'space-diagonal' of a cube measures 5.4 cm. What is its volume?

6. The total surface area of a cube is 174 cm^2. Find
   (a) its edge-length (b) its volume.

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7. A sphere has a diameter of 17 cm, what is its volume?

8. Find the surface area of a sphere having a radius of 6.3 cm.

9. The surface area of a sphere is 260 cm^2. What is its
   (a) radius (b) volume?

10. What is the total surface area of a hemisphere which has a radius of 4.3 cm?

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11. The volume of a cube is 58 cm^3. Calculate the length of one edge of the cube.

12. Find the total surface area of a cube having a volume 17.3 cm^3.

13. For a cube of volume 87 cm^3, find the length of its 'solid diagonal'.

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14. The volume of a sphere is 200 cm^3. What is its diameter?

15. A hemi-sphere has a volume of 176 cm^3. Find its radius.

16. Calculate the volume of a hemi-sphere which a total surface area of 54 cm^2.

---

17. A cube and a sphere have the same volume. The edge-length of the cube is 4.6 cm. What is the radius of the sphere?

18. A cube has a volume of 250 cm^3. This cube is to be machined off so as to leave the largest possible sphere.
    What will be the volume of the final sphere?

19. A metal cube of edge-length 2.4 cm is to be melted sown and re-cast into the shape of a hemi-sphere. What will be the diameter of the hemi-sphere?

20. A sphere of diameter 9.8 cm is to be melted down and re-cast into three identical cubes. Calculate the edge-lengths of those cubes.

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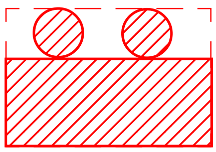
Practical Cylindricals

1	Number of can		
2	Diameter (*d*)		cm
3	Height or length (*h*)		cm
4	Curved surface area		cm^2
5	Total area of two ends		cm^2
6	Total area of sheet metal used to make complete can, neglecting any allowance for seams.		cm^2
7	Overall dimensions of rectangle needed to make the can. 	length	cm
		width	cm
8	Area of rectangle wasted in making one can		cm^2
9	Percentage of rectangle wasted in making one can		%
10	Volume of one can (interior)		cm^3
11	Capacity of can (1000 cm^3 $\equiv$ 1 litre		litres
12	Number of cans-full needed to measure out 1 litre		
13	Dimensions of a box needed to hold 24 cans when packed 3 by 4 to a layer and 2 layers high	length	cm
		width	cm
		height	cm
14	Volume of the box needed to hold 24 cans		cm^3
15	Volume of the box actually occupied by the cans		cm^3
16	Percentage of box which is 'empty'		%

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In all of these questions, the word ‘cylinder’ is to be taken as meaning a ‘right circular cylinder’.

1. For a cylinder having a radius of 3 cm and a height of 7 cm find
   (a) its volume (b) its curved surface area

2. Calculate the total surface area of a cylinder which has a diameter of 5.5 cm and a height of 8.5 cm.

3. The area of one end of a cylinder is 90 cm^2 and its height is 15 cm. What is its volume?

4. A cylinder has a volume of 500 cm^3 and the area of one of its ends is 45 cm^2. Find
   (a) its height (b) its total surface area.

5. The volume of a cylinder is 600 cm^3 and it has a height of 10.5 cm. What is its curved surface area?

6. Find the total surface area of a cylinder which has a radius of 2.7 cm and a volume of 175 cm^3.

7. What is the volume of a cylinder having a curved surface area of 250 cm^2 and a diameter of 11.3 cm?

8. A cylinder has a curved surface area of 180 cm^2 and a radius of 3.3 cm. Find its total surface area.

9. The height of a cylinder is 9.8 cm and its curved surface area is 200 cm^2. What is its diameter?

10. Calculate the height of a cylinder which has an end whose area is 50 cm^2, and which also has a curved surface area of 750 cm^2.

11. A cylinder has a diameter of 6.1 cm and a total surface area of 250 cm^2. Find
    (a) its height (b) its volume.

12. What is the volume of a cylinder which has a total surface area of 350 cm^2 and a curved surface area of 250 cm^2?

13. The volume of a cylinder is 250 cm^3 and it has a curved surface area of 150 cm^2. Calculate its height.

14. What is the volume of a cylinder having a height of 8.4 cm and a total surface area of 200 cm^2?

15. Find the diameter of a cylinder which has a volume of 115 cm^3 and a total surface area of 140 cm^2. *(More than one answer is possible.)*

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