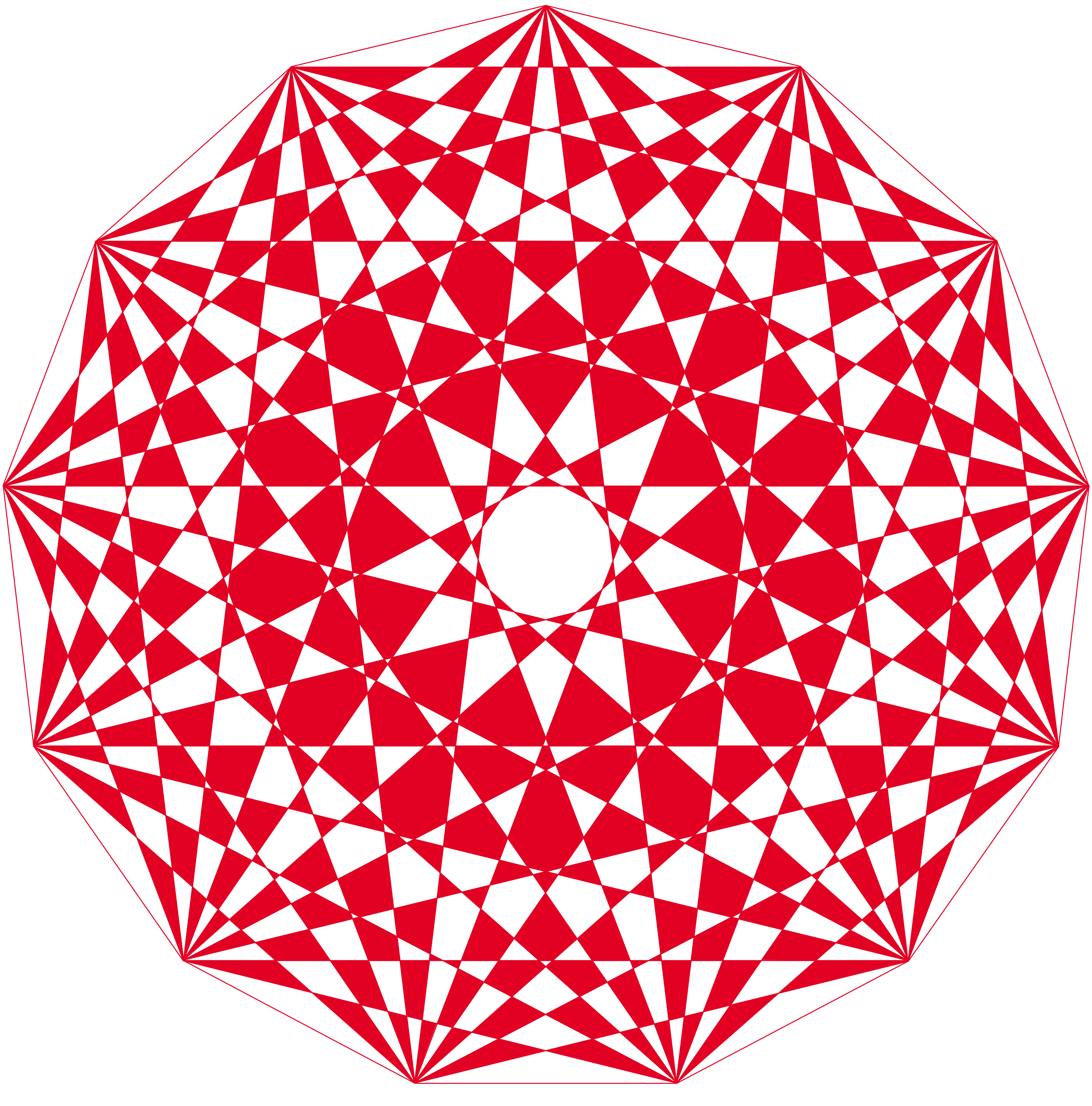
Geometric Patterns



Geometric Patterns

Introduction

In working with shapes, whether mathematically or artistically, it is necessary to have a good feeling for how more complex shapes can be made from simpler shapes. Their analysis or deconstruction is important to understanding them and to their creation. This unit is intended to help foster that sort of insight by means of some pleasurable practice in the construction of geometric patterns.

No instructions are given on the sheets themselves so that they may be used in any way deemed suitable. Notes are given here which, together with a look at the material available, will provide some guidance as to how a suitable package can be put together. It is certainly not envisaged that any one pupil would encounter every sheet.

As well as various blank "grids" provided at the back of this unit, other suitable sheets can be found from the ***trol*** menu under the headings of 'Lined Grids' and 'Dotted grids'.

One thing which needs to be borne in mind when planning this work, is the balance needed between the various aspects of the overall activity. Copying is a part of it, but should not become the whole of it, and there should be a development into exploration and creation. Even colouring-in has a part to play in encouraging the activity overall, but must not be allowed to consume an undue proportion of the time.

Geometric Patterns 1 ~ Border Patterns

This is a very good introductiory activity since it involves no more than counting squares but, for some, it does present difficulties in visually carrying something across from one sheet to another, especially when crossings are involved. A good size of square to use is 5 or 7 mm. A dotted grid is better than a lined one, since then there are no obtrusive lines in the finished drawing.

Once a straight length of border has been made it could prompt the question "How do we turn a corner with it?" This has been done in the bottom right-hand corner (No. 8) of the sheet. Notice that what happens in the detail of the corner is different from the straight border itself, but retains the style without breaking any of the lines. Going round a corner can be difficult. One technique is to draw the the straight border first and make a tracing of it. Then give the tracing a quarter-turn relative to the original (which way?) and move it around - in and out, up and down, but always keeping it at right-angles to the original - until a position is found that seems to offer a good way of allowing the lines to 'flow' between the two. There is always more than one possibility. Notice that fixing the design of a corner also determines which is the 'inside' of the border and which the 'outside'. 5 mm squared-paper is a good size to use.

Geometric Patterns 2 ~ Tile Patterns

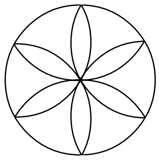
This would also be a suitable introduction as it is again based on squares. A 10 mm square is a suitable size here. One drawback is the appearance of the grid-lines in the finished drawing. Some "rubbing-out" is implied, but this is not possible with a pre-printed grid. One way of improving on this is to use a lined-grid for the initial construction work and do the final tile on a dotted-grid. This sheet could be used as a basis for launching a small project.

Geometric Patterns 3 to 6

All of these are concerned with the basic idea of dividing a circle into a number of equal parts, a topic which has an important place in much geometrical work, called **cyclotomy**.

> **cyclotomy** is the topic concerned with dividing the circumference of a circle into equal parts, using geometrical constructions which involve the use of only a pair of compasses and a straight-edge; the measuring of lengths or angles is not allowed. The early Greek mathematicians knew it was possible for cases where the number of divisions was 2^n, 3 or 5 and all other numbers obtained by multiplying any *two* of those together. So it was known to be possible for 2, 3, 4, 5, 6, 8, 10, 12, 15, … divisions. The problem of whether other divisions might be possible was unresolved until Gauss (who started on the problem when aged 19) proved that it was possible to construct $(2^2)^n + 1$ divisions provided only that the expression yielded a prime. That added 17, 257 and 65537 (n = 2, 3, 4) to the list.

Now it is one thing to know it can be done, but quite another to actually do it, and the constructions are so involved that they are useless for all practical drawing purposes. Also, why not use all the tools at our disposal?



The easiest and most well-known of the circle divisions is into 6 equal parts by using the radius of the circle itself (that is the unaltered setting of the compass), and the "flower pattern", shown on the right, made by the 6 intersecting arcs must be the most widely drawn pattern of all, as it is nearly always seen by those practising the skill of using a compass. Just marking the off the circumference into 6 equal parts and joining them up in order to make a regular hexagon should follow.

Having got the initial idea of using compasses to step-off the distances around the circle, it is a logical move to think about stepping-off other distances. Suppose we wish to divide the circle into 5 equal parts. Since 5 is less than 6 the steps will need to be bigger (won't they? - try asking first). How much bigger? There is only one way to find out - do it. A period of 'trial and adjustment' now starts, but it needs to be done in an orderly manner. Patience is needed. Open out the compasses a little, make a starting mark, and step around the circle making as faint a mark as possible, counting as you go. The 5th mark should coincide with the starting mark. When it doesn't, use the difference to adjust the setting on the compass, don't just yank at them savagely! First off, note the direction they have to be moved. If the distance is short they need to be opened, if it went past the starting mark then they have to be closed. But further than that, we can estimate the amount by which they need to be altered, it is one-fifth of the error. If it all sounds very simple and obvious, just wait until your class tries out this technique!

In case this seems unnecessarily complicated, it isn't. It has the merits of being easily understood, applicable to all cases (whatever the number of divisions) and extremely accurate. Of course that amount of accuracy may not be necessary but, if it is, it cannot be achieved any other way - unless you can measure angles to at least 0**.**1 of a degree. A test of accuracy would be the requirement to draw the 13-point circle and all its diagonals, as on the front page of this unit, and see that all those small triangles appeared. It perhaps needs to be said that such work requires very good drawing instruments, especially a pair of needle-point dividers with a fine-adjustment facility, if it is to be done by hand. The usual basic school compass with a blunt pencil does not rate highly for doing the best work. Of course, a good computer drawing program, with its ability to draw angles to an accuracy of 0.01° makes it much easier!

Having 'mastered' the technique, or by using the appropriate templates from those given at the back of this unit, some work based on sheets 3 to 6 could be done.

Polygons

It will be clear that polygons underlie much of this work, if only implicitly, but it should be made explicit whenever opportunity offers. Attaching the correct names to the various polygons as well as the other technical words associated with them would be a good starting-point.

The last page in this unit (Polygons ~ Vocabulary and Data) would help in this, and all pupils could benefit from having a copy of that page to keep in their notebooks. The table on that sheet could be used in several ways, one of the more obvious being for some formal mensuration. But it can be used to find what the edge-length of the underlying polygon must be for any given number of divisions of the circle. This is the setting needed on the compasses to step around the circle. It reduces some of the work needed for the ‘trial and adjustment’ method described earlier, though it will still be necessary for the most exacting work.

Focussing upon the polygons themselves could lead to some other work. For any given polygon, how many diagonals can be drawn? The data-table does provide this information so, if that table is generally available, ask instead how many diagonals must there be in a 20-gon? This not very difficult since there is an easily seen pattern of growth, but that could lead to a request for a general formula. Then, having drawn all the diagonals, how many regions are there within the (regular) polygon? This is much harder.

Returning to the patterns. The symmetry of various patterns should be remarked upon, that is for both line- and rotational-symmetry. Also the effect that colouring can have upon those symmetries.

The last few pages contain a miscellany of examples which the more adventurous might like to look at for ideas.

Other Work and Sources

Other work which would help with the exploration and understanding of shapes would be

Tangrams, **Tessellations**, **Pentominoes**,

each can be found in the ***trol*** menu under 'Other Activities'.

additionally, there is some related work, in a different context, provided under

Calendar Models (to be found at the top of the ***trol*** menu)

and, in there, the Wall Calendars identified as **Geometric Patterns**

The greatest form of Geometric Patterns is to be found in Islamic Art, and some representation of that ought to be available for all to see. There are many books on it. A highly recommended one is

Geometric Concepts in Islamic Art by Issam El-Said and Ayse Parman

ISBN 0 905035 03 8

It generously illustrated not only with pictures of the actual art in its place, but also with clear diagrams showing how the patterns are made. There should be a copy in any half-decent library either at school or departmental level. First published in 1976, it is still in print and not at all expensive considering what it offers. Great value.

A smaller book, but very useful to have available in the classroom, is

Geometric Patterns from Roman Mosaics by Robert Field

ISBN 0 906212 63 4

This one is published by those excellent people at Tarquin Publications

There is quite a lot on the Web. Call up a Search-engine, **www.google.com** is recommended, and give it "islamic patterns" to get over a thousand possibilities. (note the inverted commas are needed or else you will get nearer one-hundred thousand!). Unfortunately many of them are only concerned with selling books, but that still leaves plenty of sites which show beautifully detailed pictures of this type of work, which does mean they are rather large files if down-loading times are a consideration. Several schools have put up examples of work done by pupils.

Geometric Patterns ~ 1

Border Patterns

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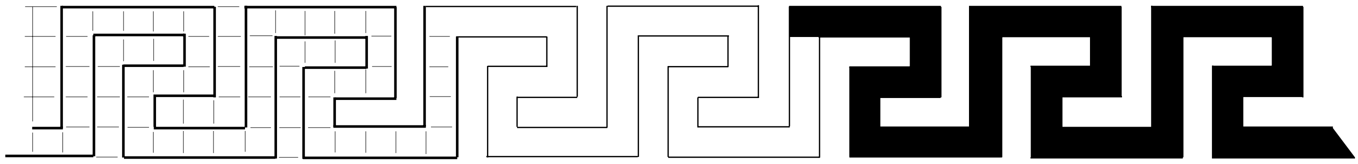

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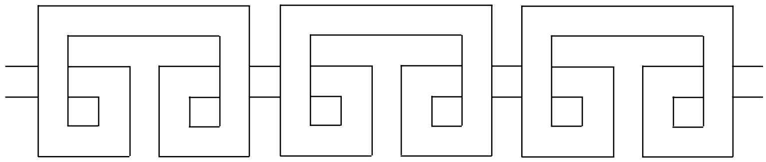

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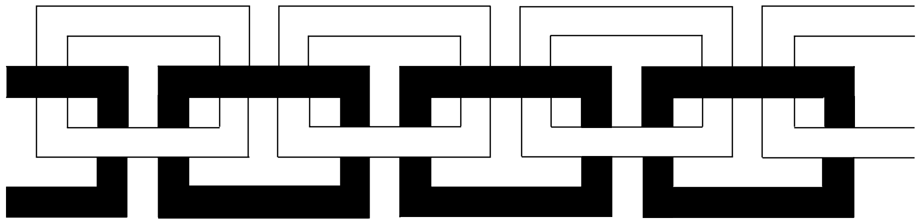

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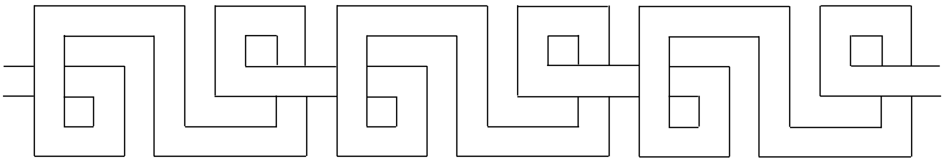

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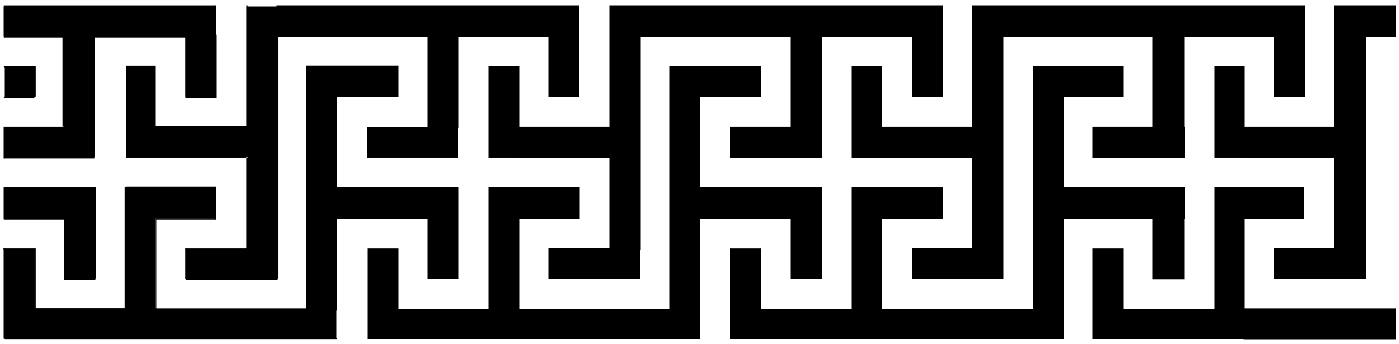

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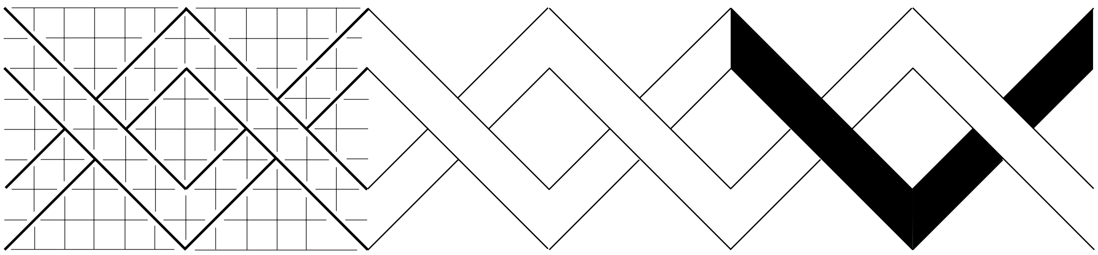

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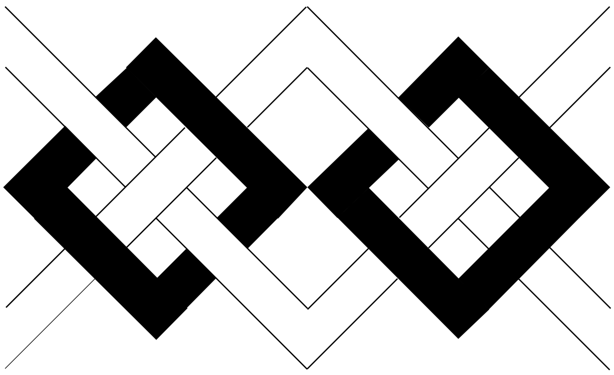

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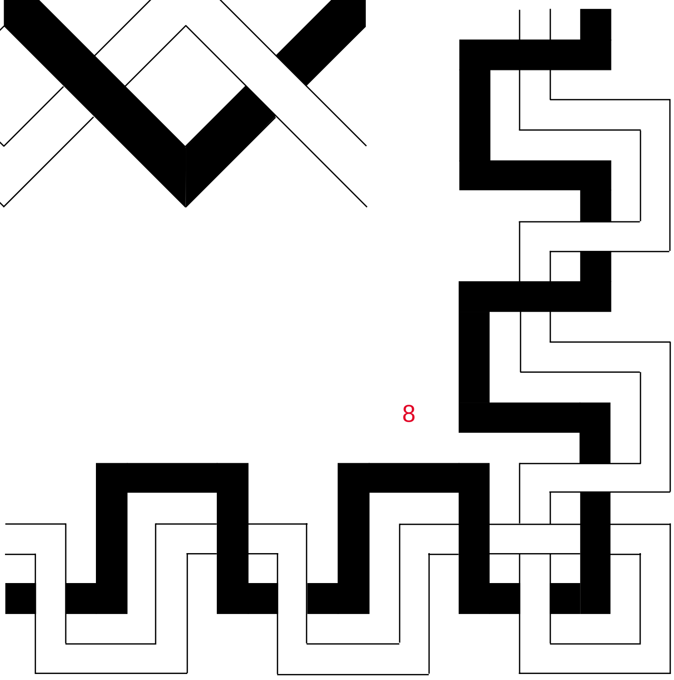
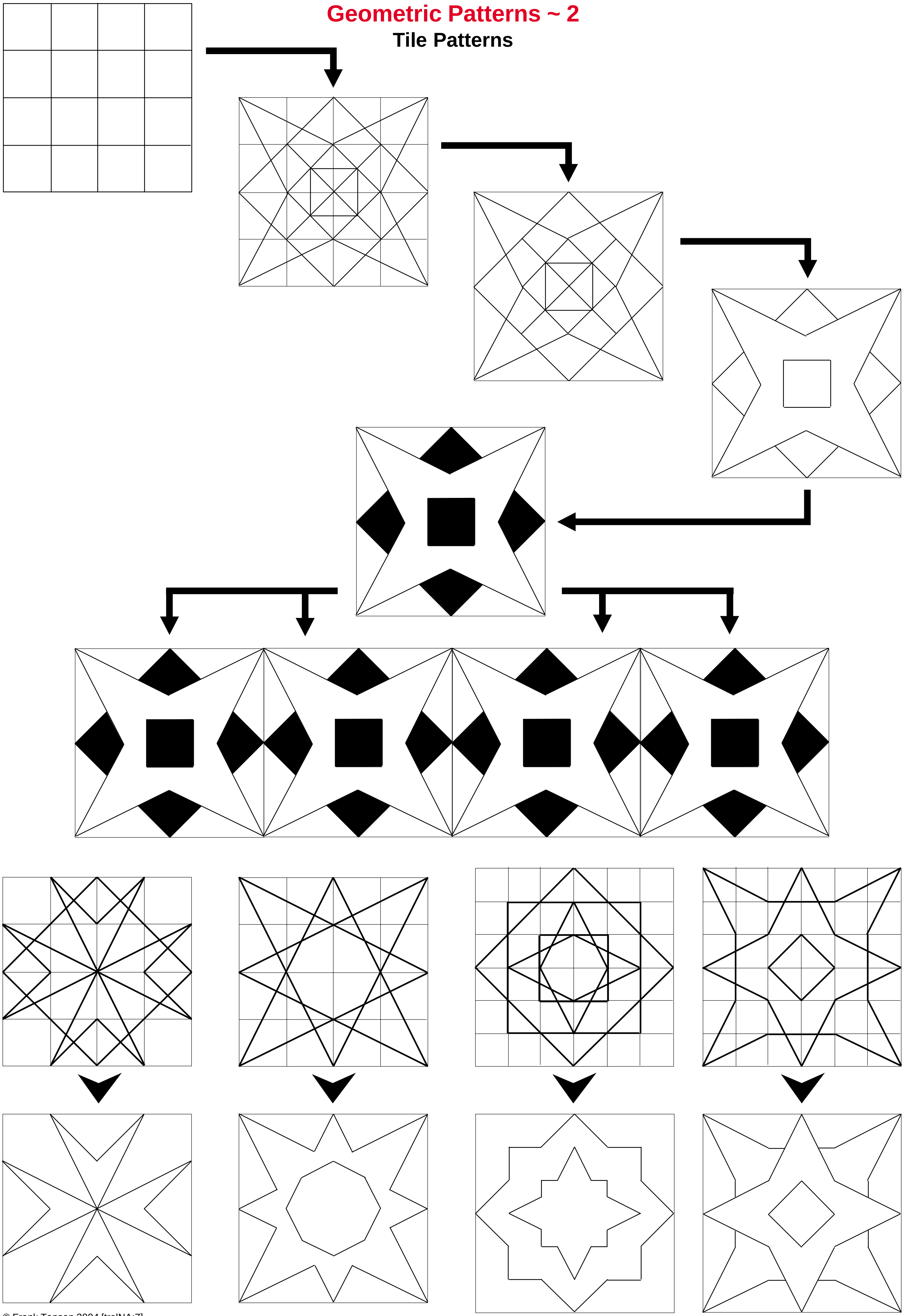


Geometric Patterns ~ 2

Tile Patterns



Geometric Patterns ~ 3

The 6 - point circle

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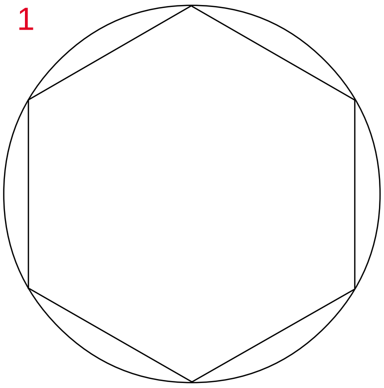

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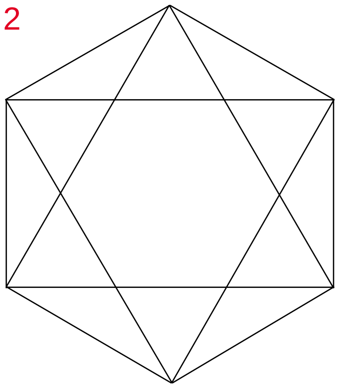

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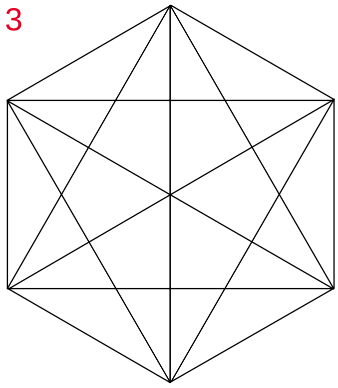

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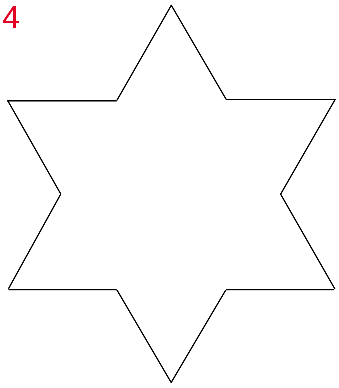

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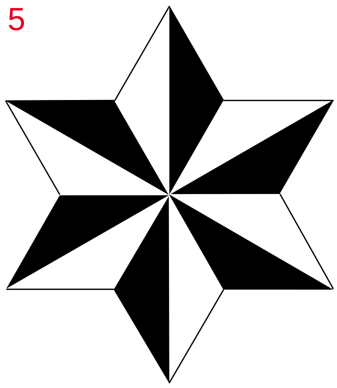

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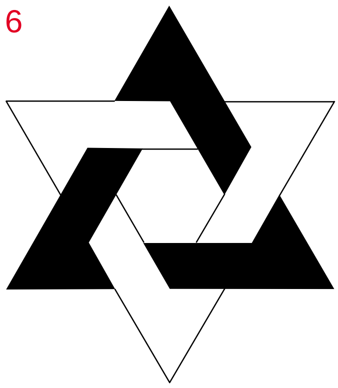

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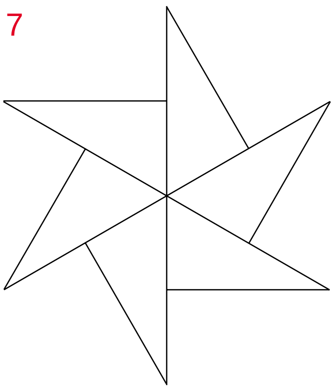

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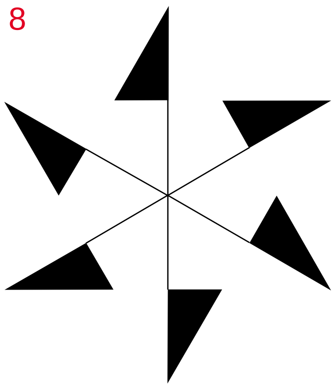

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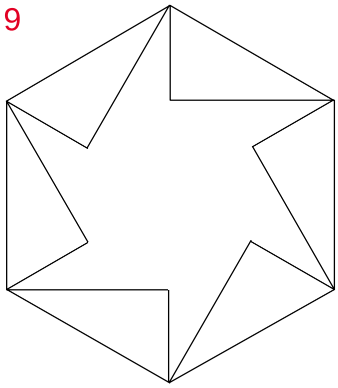

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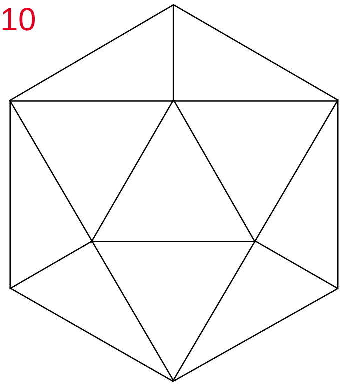

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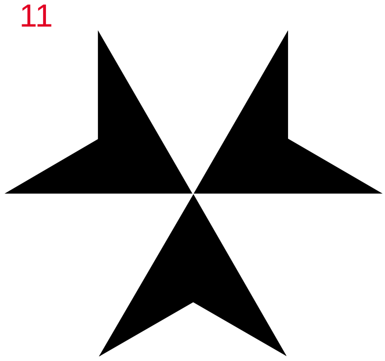

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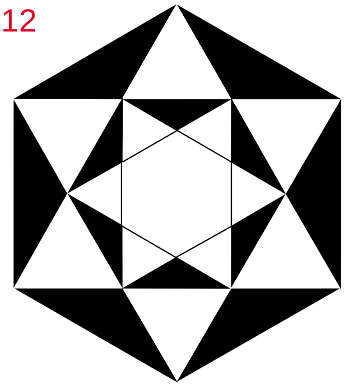

Geometric Patterns ~ 4

The 8 - point circle

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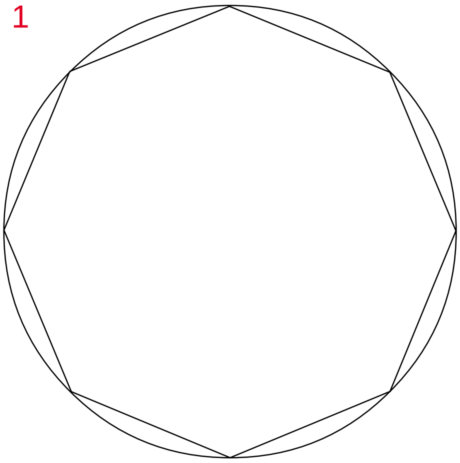

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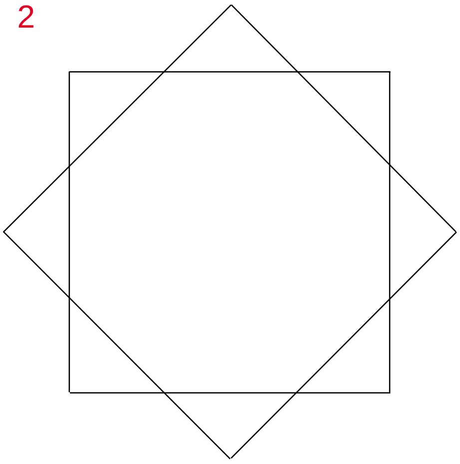

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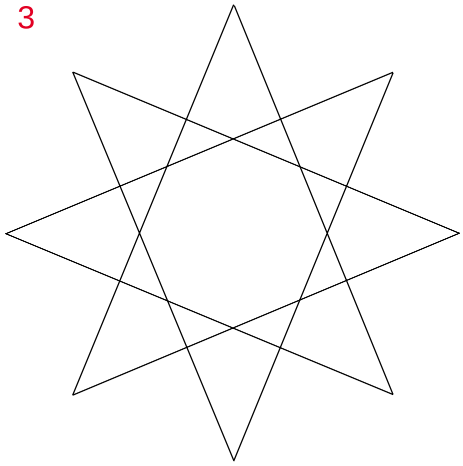

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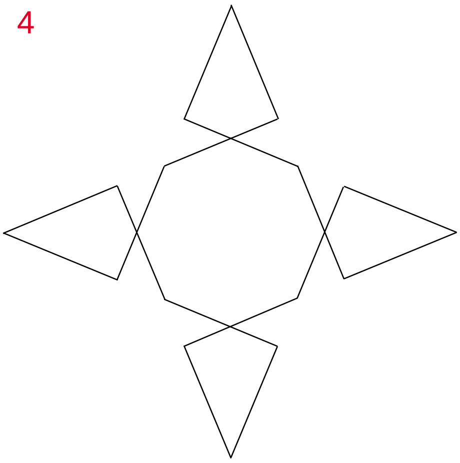

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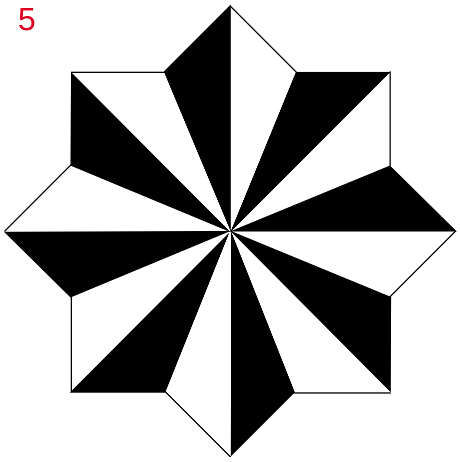

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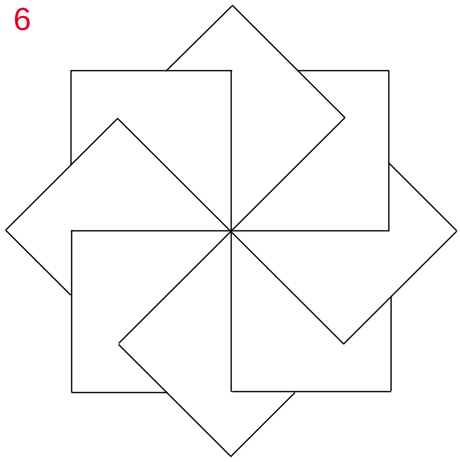

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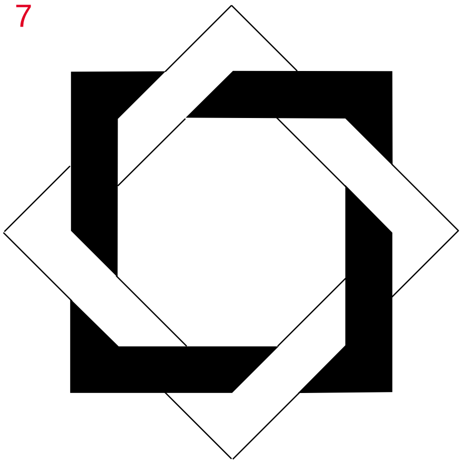

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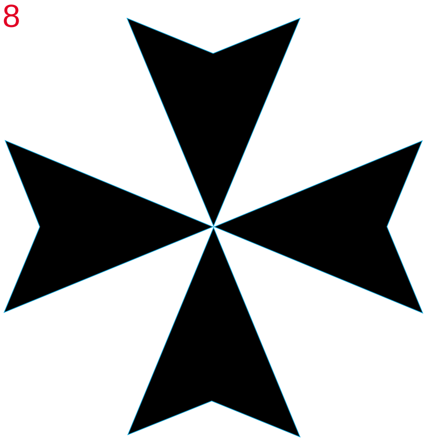

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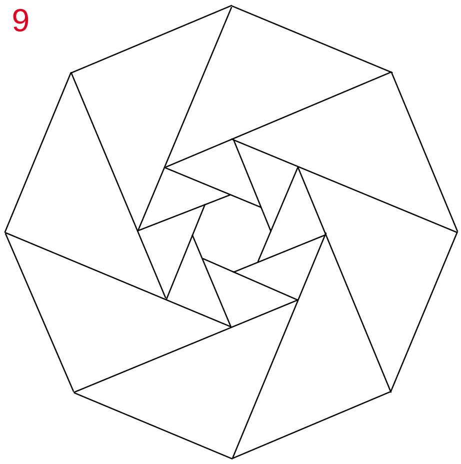

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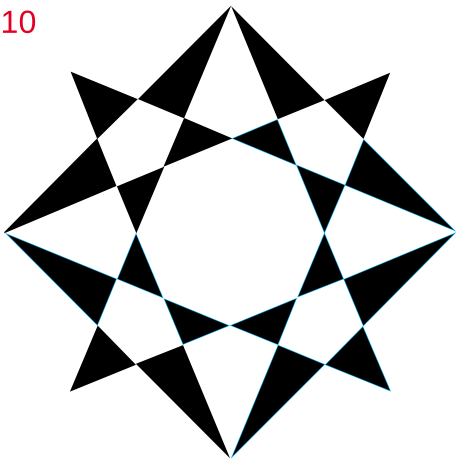

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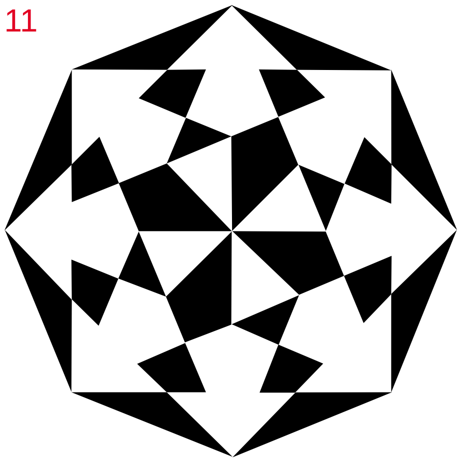

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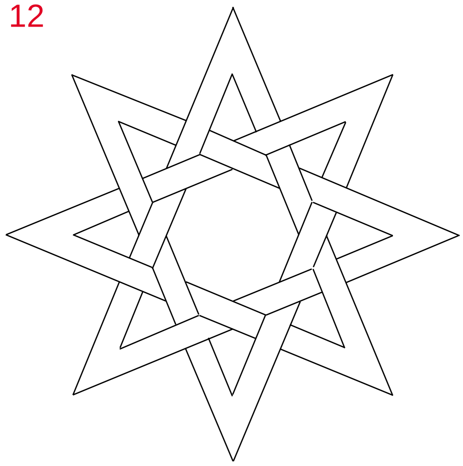

Geometric Patterns ~ 5

The 5/10 - point circle

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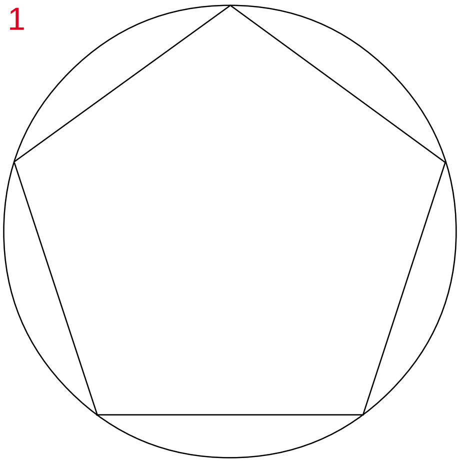

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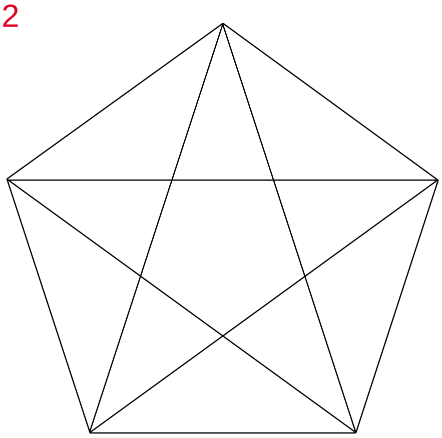

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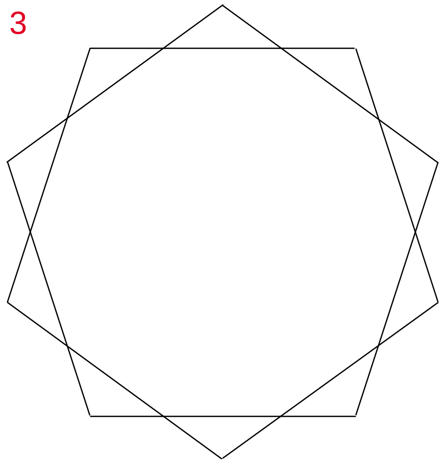

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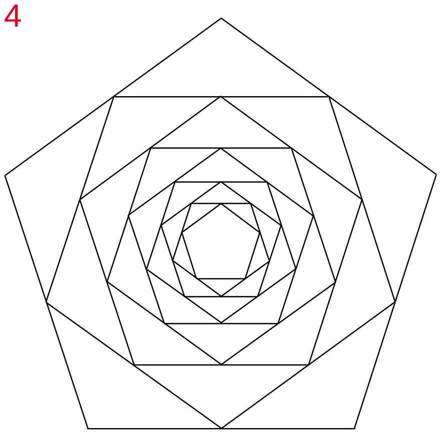

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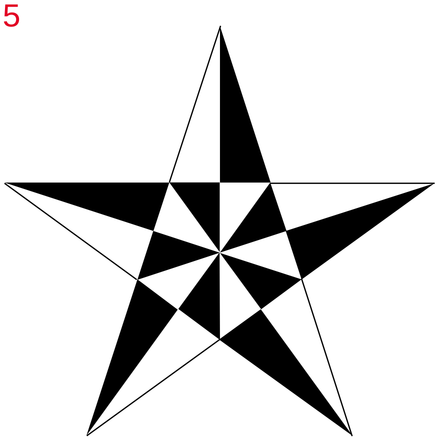

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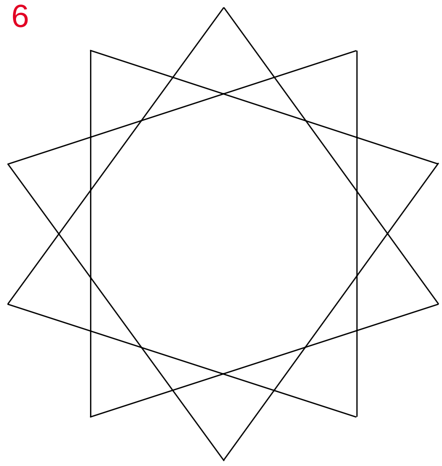

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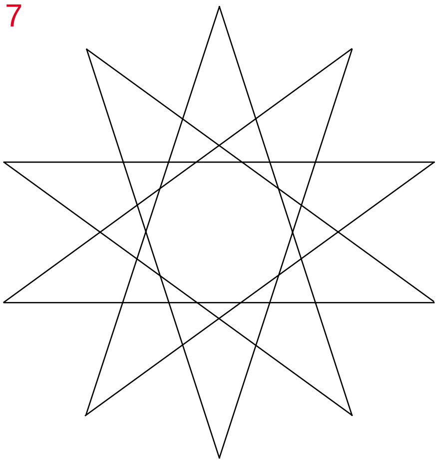

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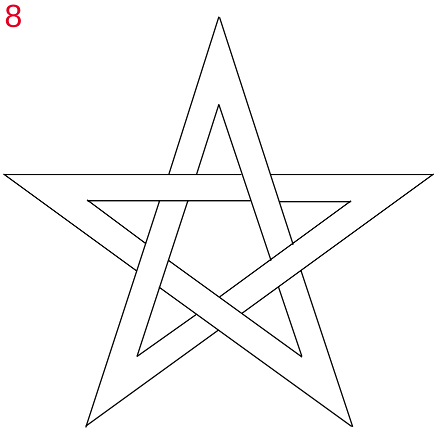

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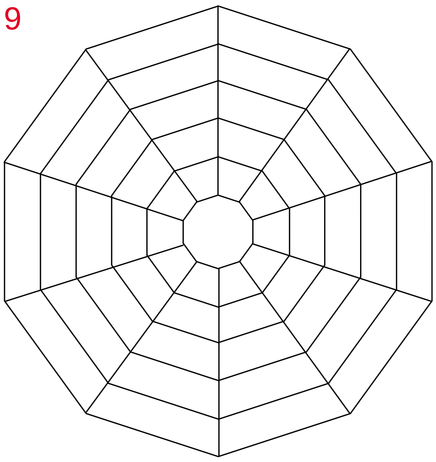

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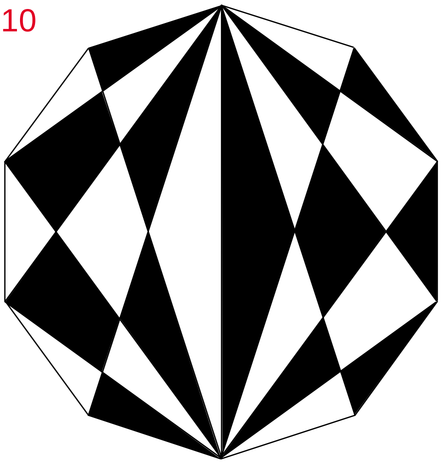

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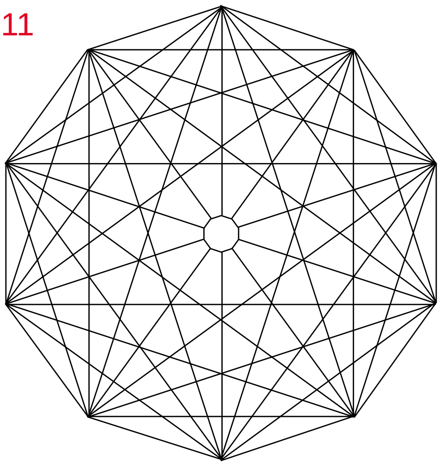

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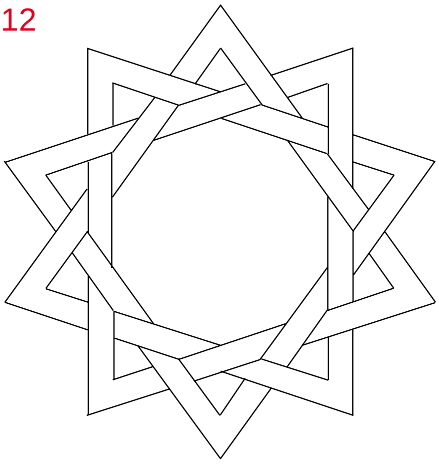

Geometric Patterns ~ 6

The 12 - point circle

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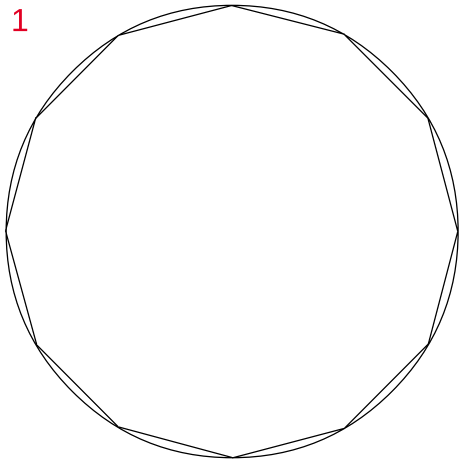

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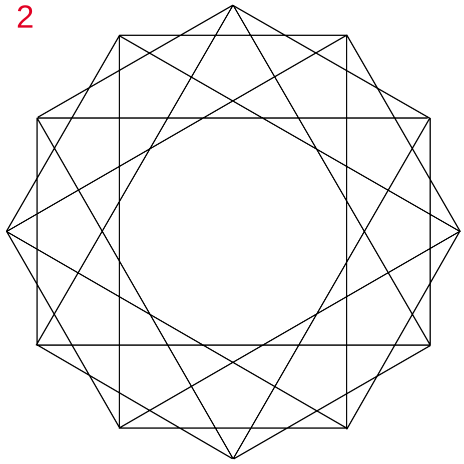

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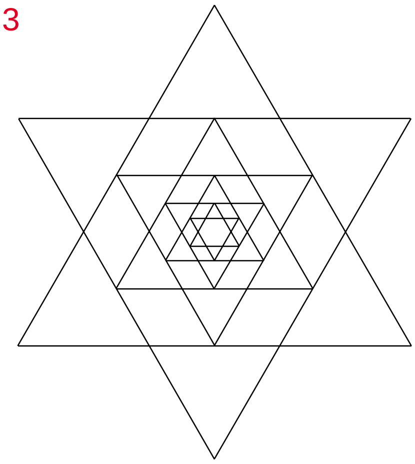

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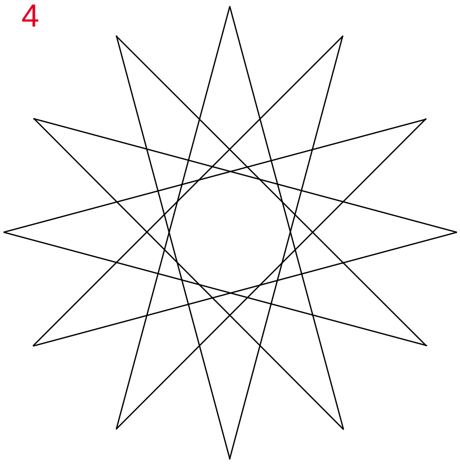

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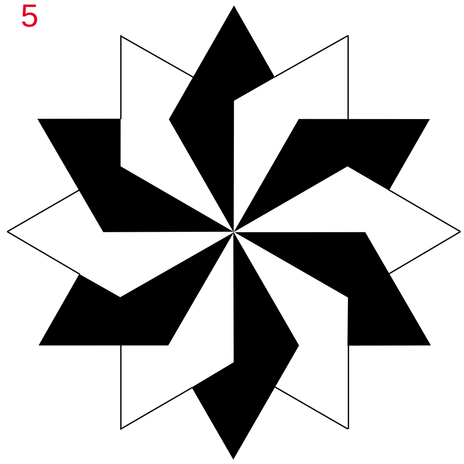

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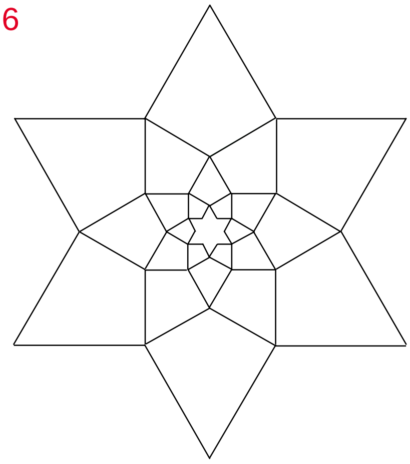

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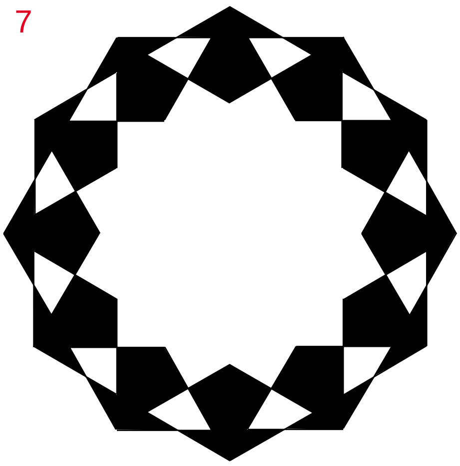

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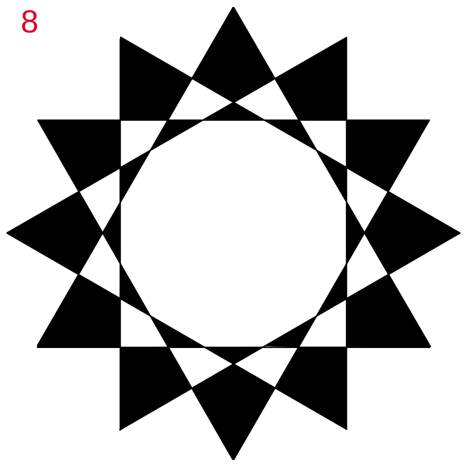

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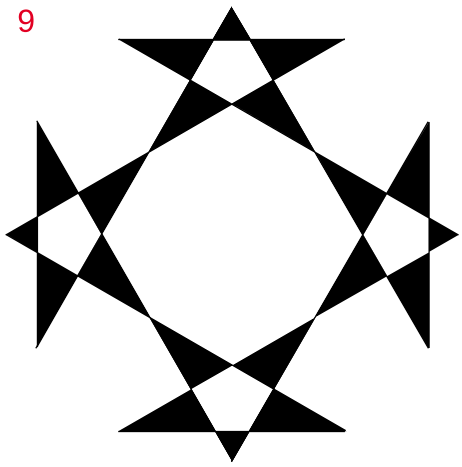

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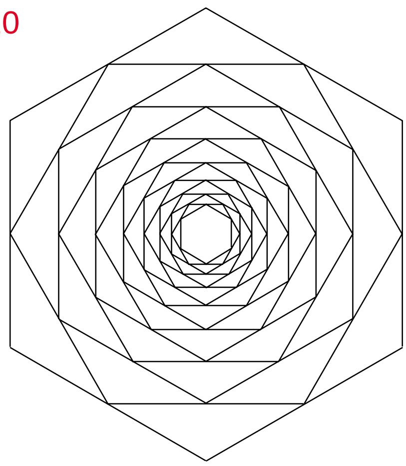

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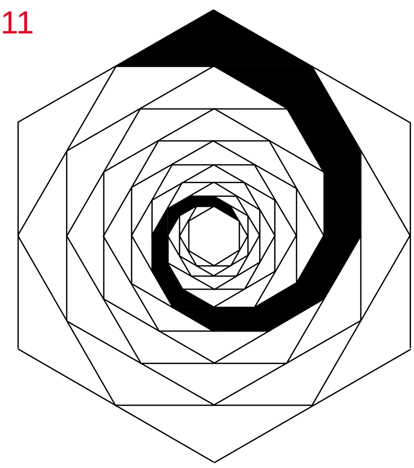

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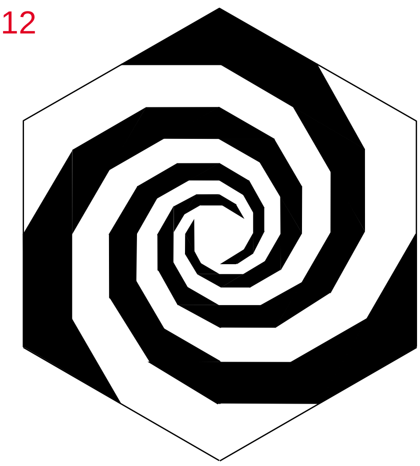
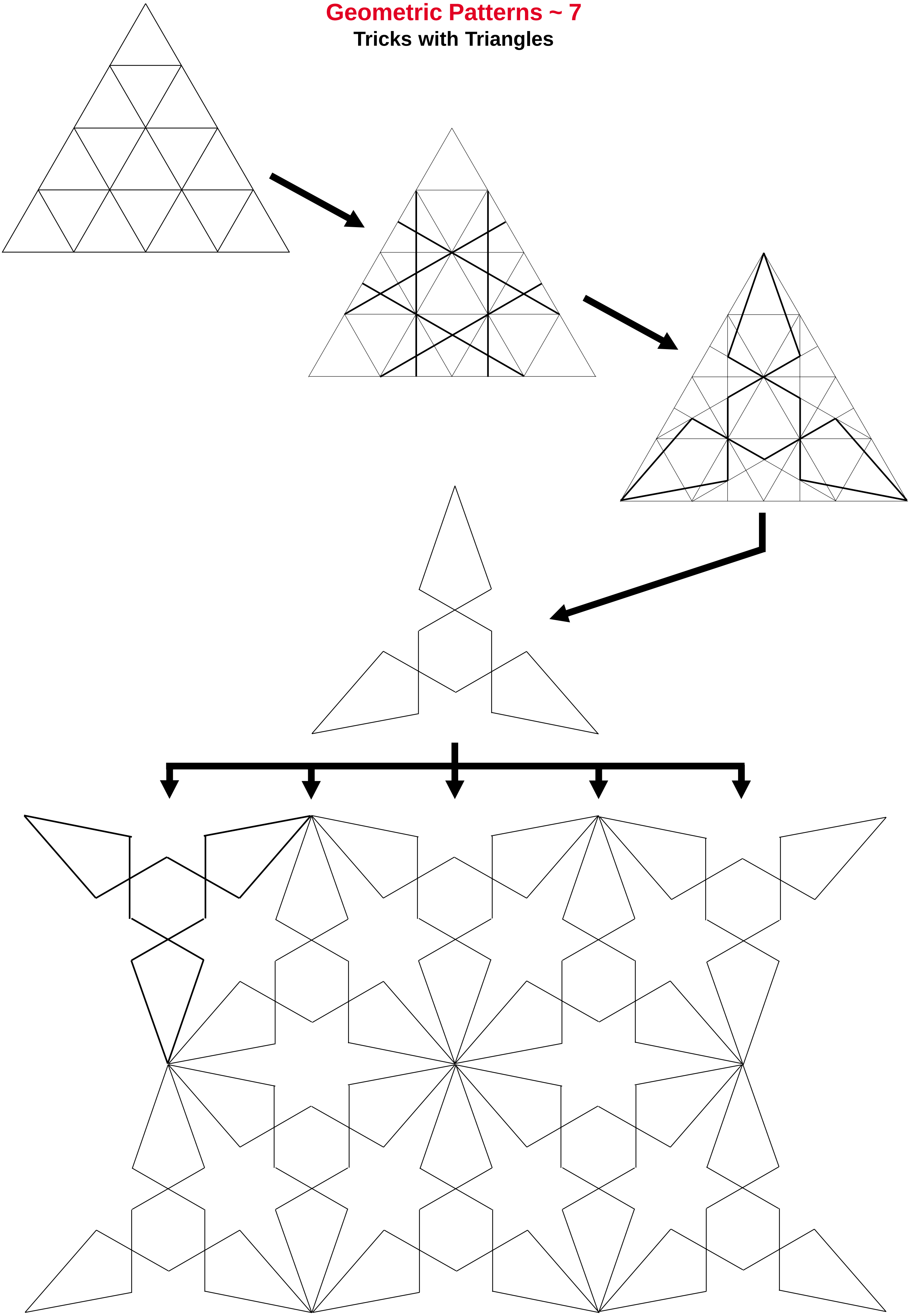
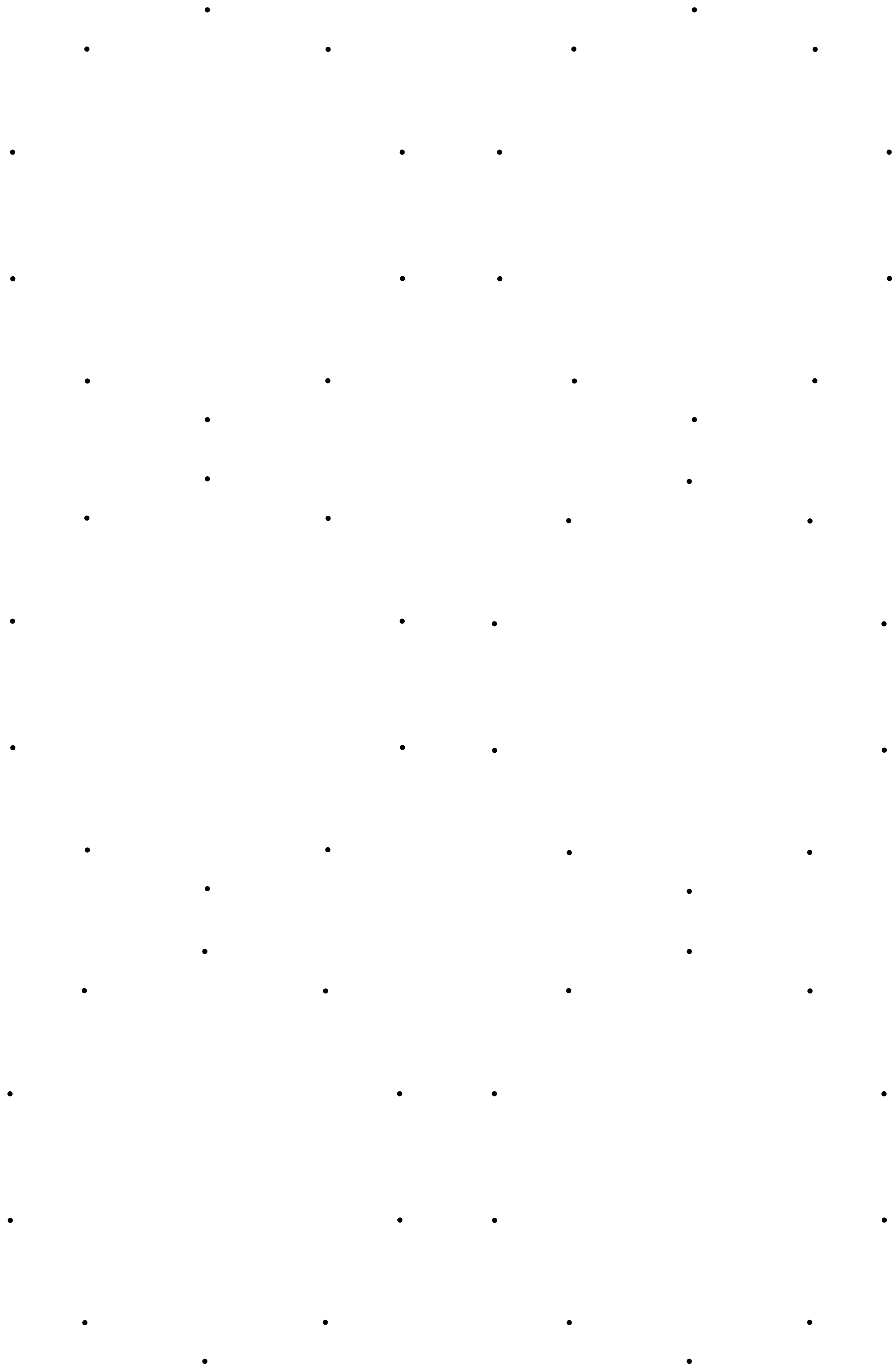
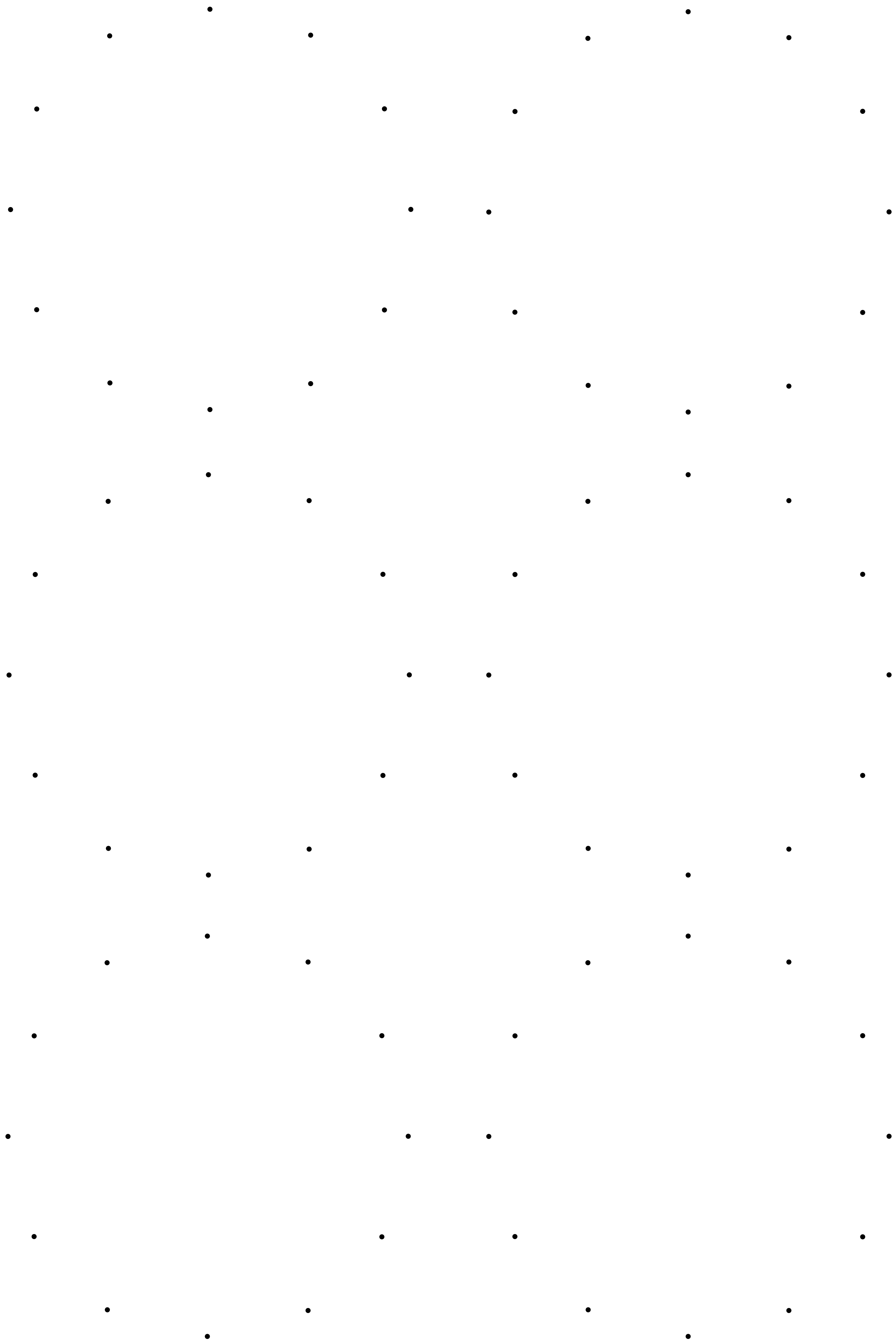


Geometric Patterns ~ 7

Tricks with Triangles







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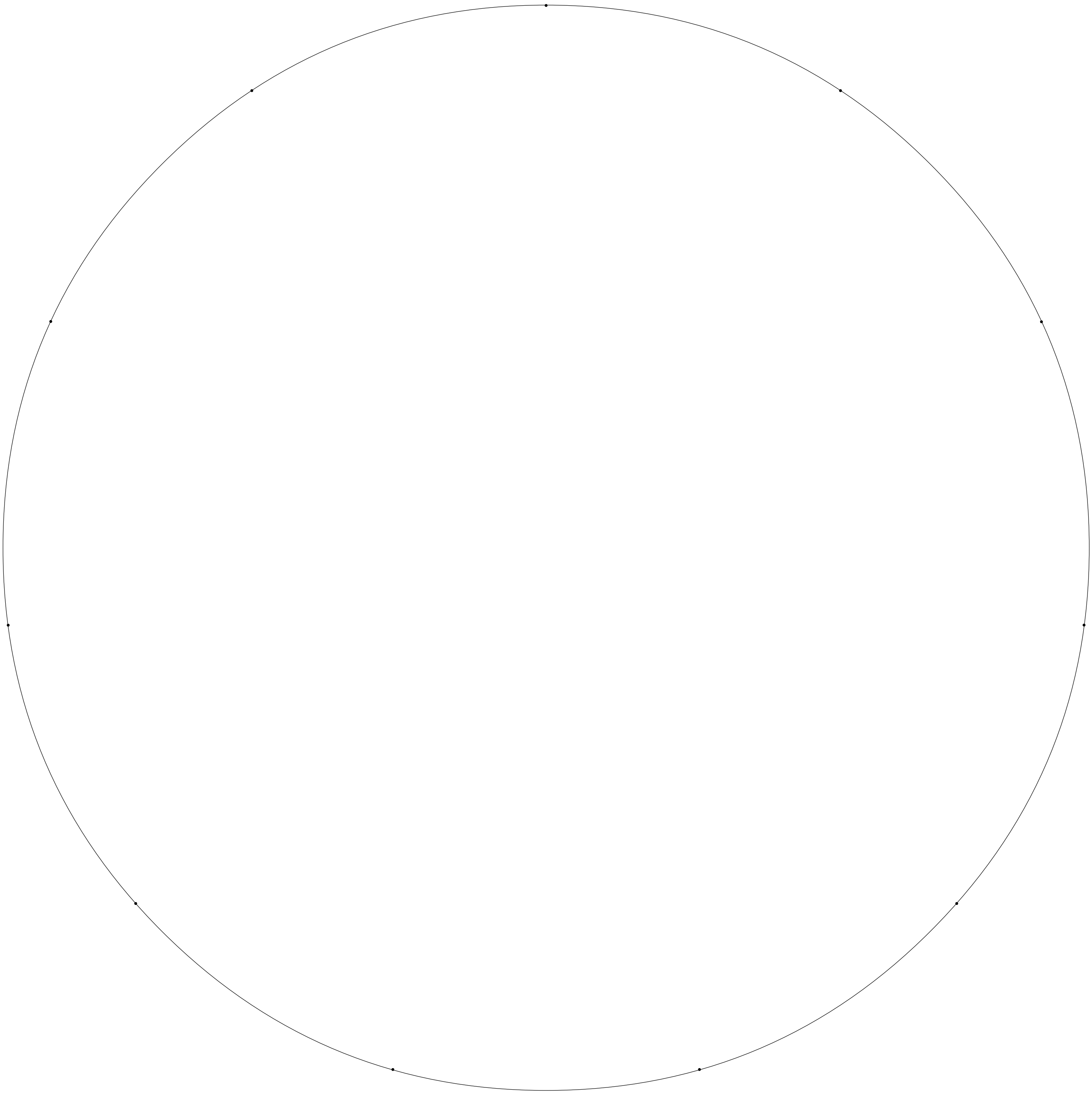

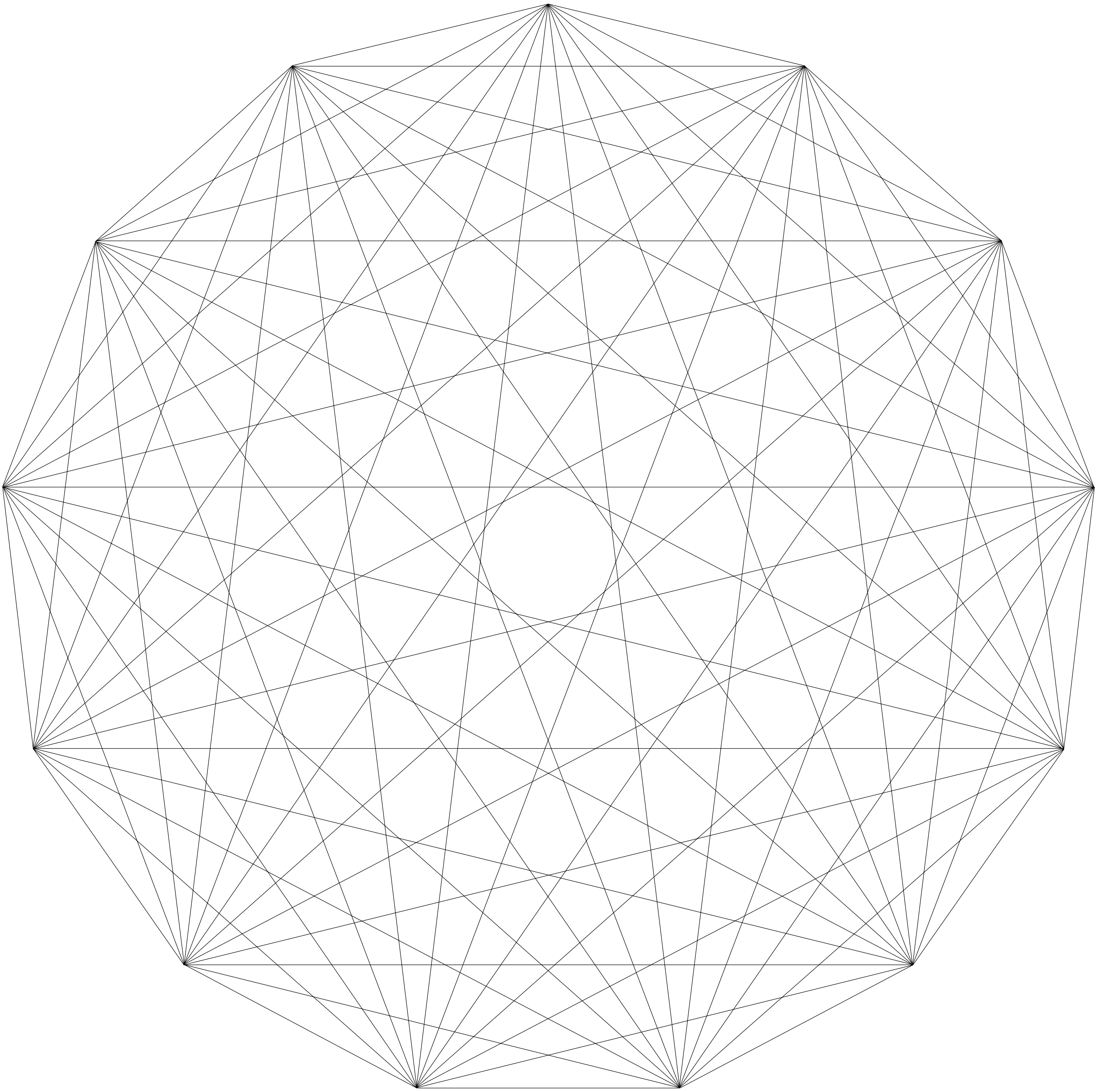

Polygons ~ Vocabulary and Data

polygon A polygon is a plane (= *flat*) shape completely enclosed by three or more straight edges. *Usually, edges are not allowed to cross one another, and the word is not used for shapes having less than 5 edges. Polygons are named by reference to the number of edges or angles they have - see table below.*

vertex A vertex is a point where two edges of a polygon meet to form a corner.

equilateral An equilateral polygon is one whose edges are all the same length.

equiangular An equiangular polygon is one which has the same angle at every vertex.

isogon An isogon is another name for an equiangular polygon.

regular A regular polygon is one which is both equilateral and equiangular.

irregular An irregular polygon is one which is not regular

concave A concave polygon is one which has at least one vertex with an interior angle which is greater than 180°

convex A convex polygon is one in which the interior angle of every vertex is less than 180°. *All regular polygons are convex. All its diagonals lie inside the polygon.*

re-entrant A re-entrant polygon is another name for a concave polygon

diagonal A diagonal of a polygon is any straight line drawn between two vertices which are not next to each other. *A diagonal can lie outside the polygon, as in the concave case.*

circumcircle A circumcircle to any polygon is the circle drawn around the outside of that polygon which touches all of its vertices. *Since it is necessary for the circle to touch every vertex of the polygon, it is not possible to draw a circumcircle for every polygon (for example not for concave polygons), but is is always possible for regular polygons and for any triangle. The circumcircle is also the 'base' circle often used in the construction of regular polygons.*

incircle An incircle to any polygon is the circle drawn on the inside of that polygon which touches all its edges. *Each edge is a tangent to the incircle. As with the circumcircle, not every polgon has an associated incircle, but every regular polygon has one, and so do all triangles.*

star polygon A star polygon is made by joining every r^{th} vertex of a polygon having n vertices ($1 < r < n-1$); n and r having no factors in common, and $n > 4$. *They are described as star polygons{**n/r**} and also known as **n**-grams.*

Circles and their related Regular Polygons

No. of divisions	Name of polygon	Edge = r × …	Perimeter = r × …	Area = r^2 × …	I-radius = r × …	Number of Diagonals
3	**triangle**	1.7321	5.1962	1.299	0.5	0
4	**quadrilateral**	1.4142	5.6569	2	0.70711	2
5	**pentagon**	1.1756	5.8779	2.3776	0.80902	5
6	**hexagon**	1	6	2.5981	0.86603	9
7	**heptagon**	0.86777	6.0744	2.7364	0.90097	14
8	**octagon**	0.76537	6.1229	2.8284	0.92388	20
9	**nonagon***	0.68404	6.1564	2.8925	0.9397	27
10	**decagon**	0.61803	6.1803	2.9389	0.95106	35
11	**undecagon**	0.56347	6.1981	2.9735	0.95949	44
12	**dodecagon**	0.51764	6.2117	3	0.96593	54

Given the radius r of a base-circle, used to draw a regular polygon, the table enables the edge-length of the polygon, its perimeter, area and the radius of its inscribed-circle (= I-radius), all to be calculated by using the multiplier given in conjunction with the radius r.
Note the base-circle is the circumcircle to that polygon.
The number of diagonals that polygon has is also given.
Values are given to 5 significant figures, but unnecessary 0's at the end have been left off.
*a nonagon is also known as an enneagon