T-tiles

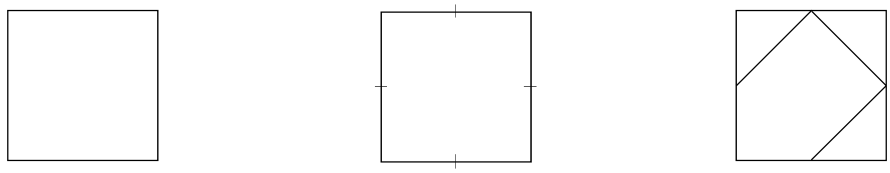
An incomplete investigation

This started life many years ago as an 'end of term' activity.
The original idea was to start with a new form of tile that was as new to the teacher as it would be to the pupils so that there were no known or ready-made solutions for anyone. Of course, the teacher had some knowledge of similar things, but would have to work as hard (harder!) as everyone else.
It provided quite a lot of work and interest and was repeated many times over several years until one day it was put into a cupboard and forgotten.
It has recently been excavated, the dust has been blown off, and the whole thing tidied up and also put into some sort of order. It is not complete by any means. Many questions are asked to which definitive answers are not known, and there are undoubtedly many more questions that could be asked.
It is a starting point, and no more than that.

Making the tiles

Start with a square. Mark off the mid-points of the 4 edges. Join these points with 3 lines in the manner shown below to create 4 areas.

1 2 3



Each of these areas has to be coloured under the following restrictions.

- Not more than 3 different colours may be used.
- Areas which touch along a common edge may not be the same colour.

An example of one such tile is

How many different tiles are possible, given that only 3 colours are available?

The word 'different' may need amplification. Tiles can be turned around but not turned over, so that 2 tiles which are mirror images of each other are classed as different.

Under the conditions 24 different tiles can be found.

An argument to support this could take these lines.

Start with a blank tile. As soon as the 'main' colour is chosen (for the big area) it clearly cannot be used again on that tile, so the 3 minor areas have to be done with the 2 remaining colours.
Using only 1 of the colours repeated on all 3 areas would create 2 different tiles.
Then it gets a little trickier. We have to use both colours at the same time to cover the 3 areas. Assigning the numbers 1 and 2 to the two different colours then we can consider the arrangements to be permutations of those numbers in 3 positions. There are 6 :–

(1, 1, 2) (1, 2, 1) (2, 1, 1) (1, 2, 2) (2, 1, 2) (2, 2, 1)

That means 6 different tiles which, plus the previous 2 makes 8
So for any given main colour exactly 8 different tiles are possible and, since there can only be 3 main colours, we have $3 \times 8 = 24$ different tiles in total.

In these matters 24 is an 'encouraging' number to find because it is so rich in factors.

Now what?

Well, make a set and play with them!

Suggestions are given on the next page as to what might be looked for.

A sheet of blank tiles (35) is provided which could be used to discover the full set.

A set of the 24 tiles already coloured is available for those not wishing to make their own.

Both of those sets are based on a 35 mm square.

There are 3 sheets of grids with mid-edge markings to facilitate the drawing of solutions. These different sheets allow for 35, 30 or 25 mm squares to be used for the record.

There are 2 sheets containing solutions to a few of the possible problems. These drawings are based on a 20 mm square. For those who have difficulty in seeing the separate tiles from which the patterns are made it may be useful to have an overlay of tracing paper with a 20 mm squared grid drawn on it.

Some problems

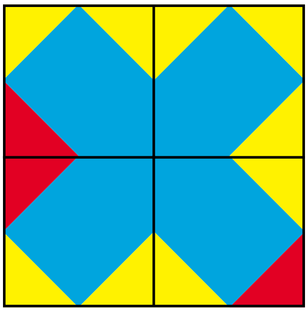
In all that follows, where tiles are being brought into contact with one another, the rule is that touching edges must match.

Arrange them in a long 'chain'. Make the chain close so that it forms a rectangle. Invent a game (like dominoes) to be played with these tiles.

Divide the full set into 3 sub-sets according to their main colour.

Split each sub-set into 2 groups of 4, and assemble each group into a cross. An example is shown on the right

In how many different ways is it possible to do this?



Make complete rectangles of various sizes (tiles by tiles)

- 6 by 2
- 8 by 2
- 4 by 3
- 4 by 4
- 5 by 4
- 6 by 4
- 8 by 3
- 5 by 5 (with a 'hole' in the middle)

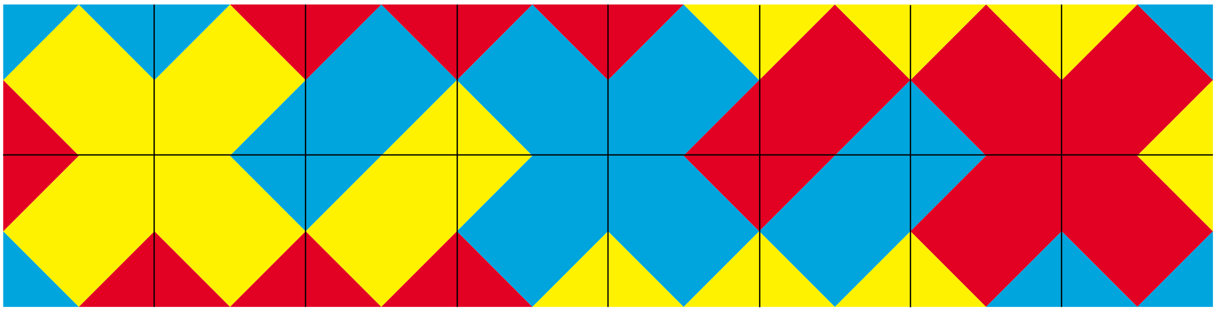
Make a complete rectangle such that its upper and lower edges match, and also its left and right-hand edges. Under those conditions what is the largest rectangle possible? Note that, using a rectangle like that as a motif, it would be possible to tesselate the infinite plane.

What is the largest possible shape having symmetry in both shape and colour?

12 of the tiles are asymmetric which must mean that they exist in 2 forms - one being a mirror image of the other. This should mean that those 12 can be assembled in such a way that the shape has a line of symmetry. Find the 12 and make such a shape. What about the other 12?

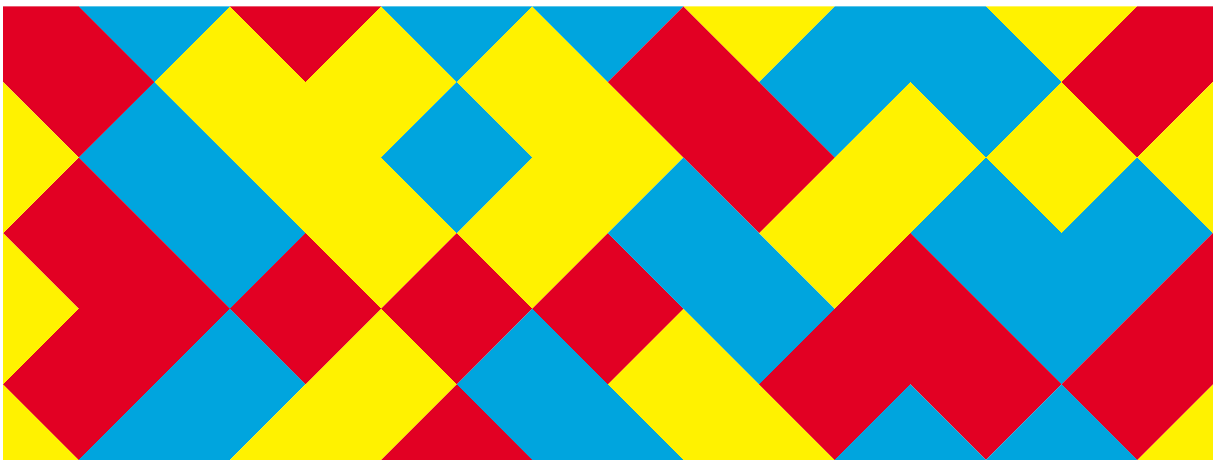
Some Patterns made with T - tiles (1)

A 6 by 2 arrangement having horizontal symmetry for both shapes and colours, and vertical symmetry for shapes only.



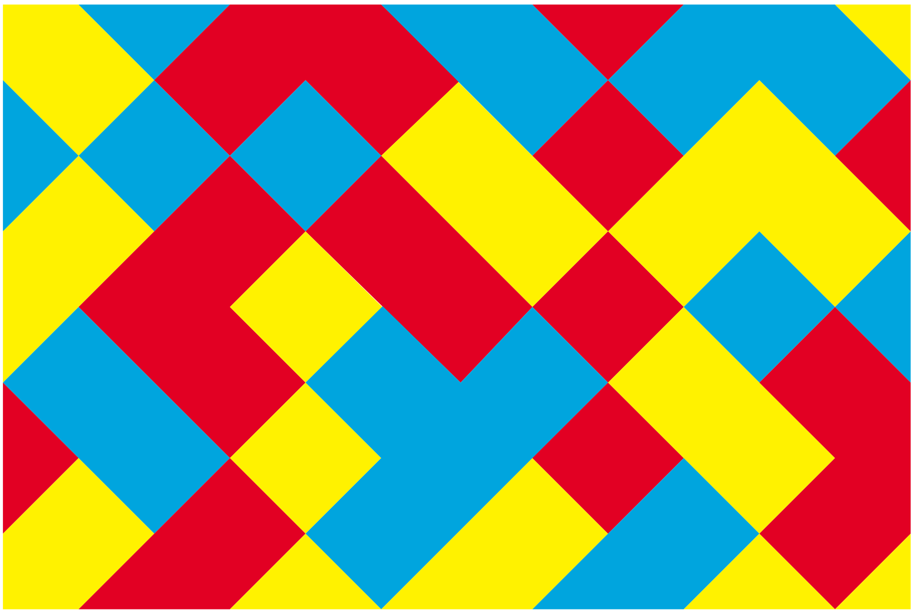
8 by 2

8 by 2

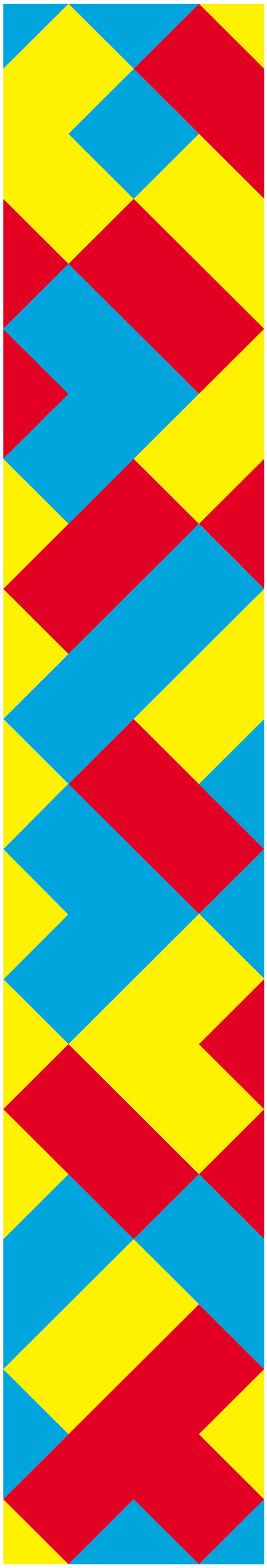


8 by 3

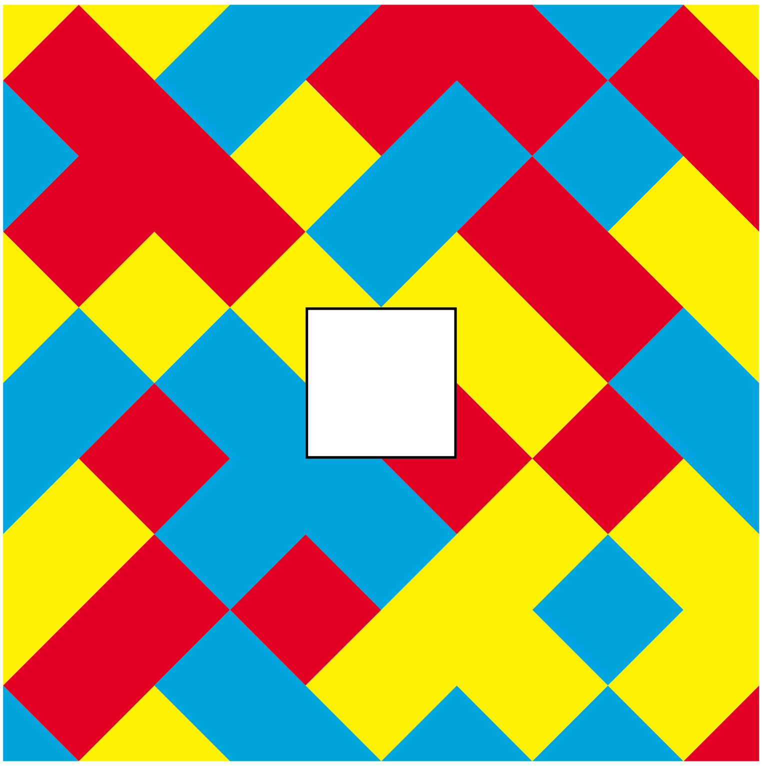
Some Patterns made with T - tiles (2)



6 by 4

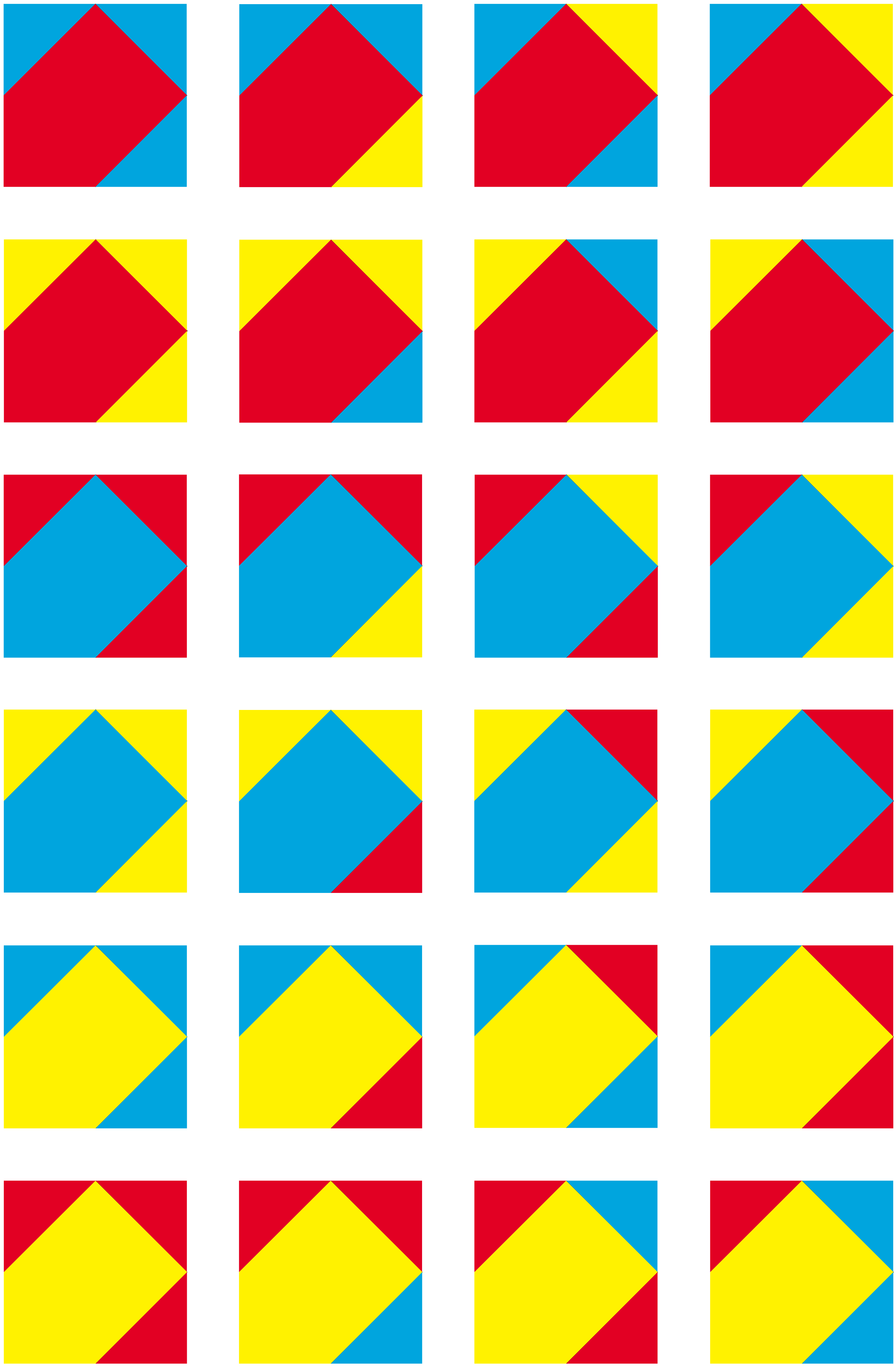


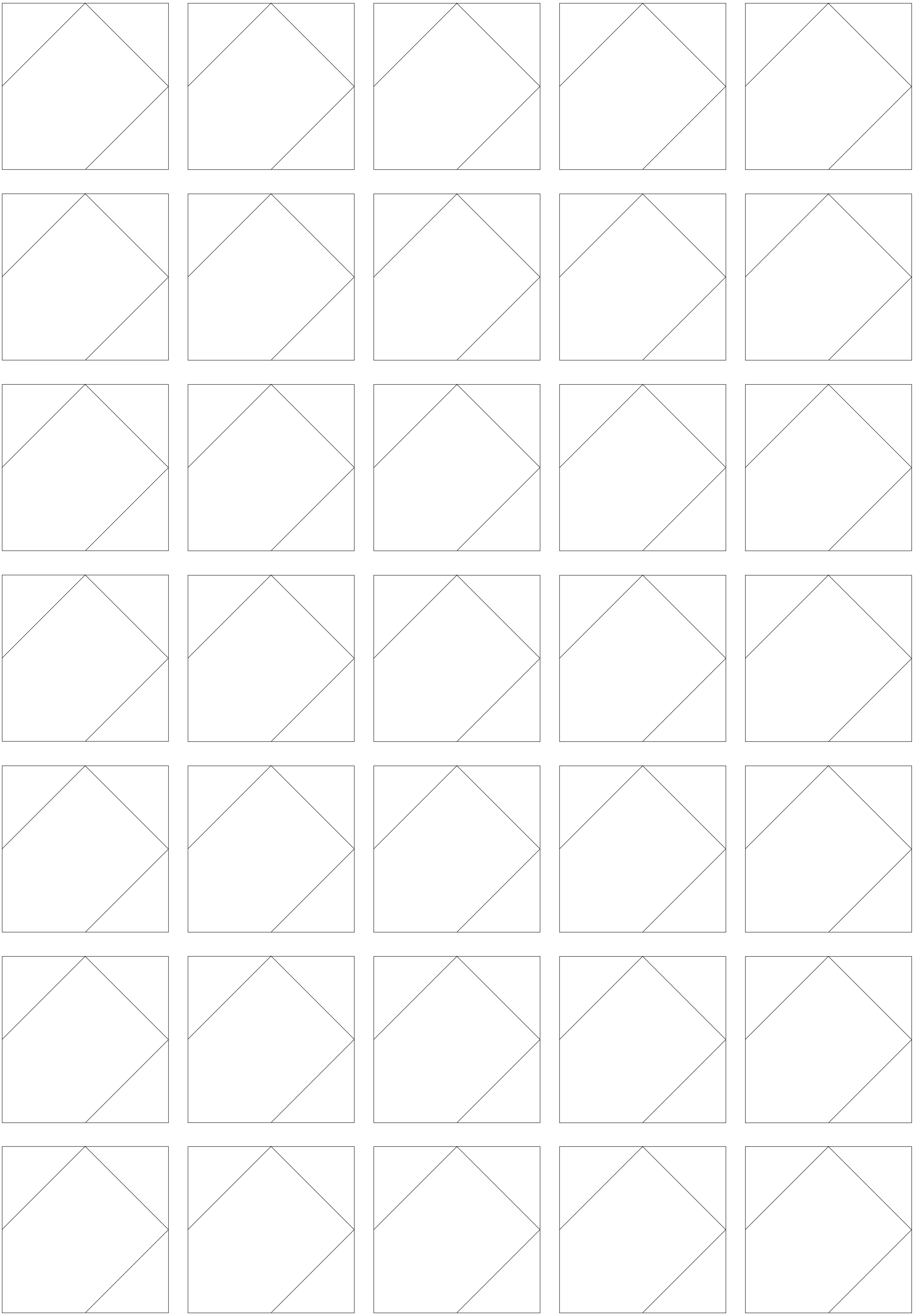
12 by 2



5 by 5 (with a hole in it)

T - tiles





35 mm

30 mm

25 mm