

# Multiplication Methods

A collection of some methods for doing multiplication that have been devised and used in previous times.

## Notes on Multiplication Methods ~ 1

All of the methods given here are written methods.

For an additional account of some mechanical methods see under 'Artifacts for Mathematics' (in the *tr01* menu) where the material for making, together with instructions for using: Napier's Rods, Genaille's Rods and a Slide Rule, are to be found.

At the back of this unit are some sheets of multiplication tables.

The first is simply eight copies of a basic 1 to 9 multiplication square, which makes it easy to provide everyone with their own personal copy when those facts are needed. Needing to "know their tables" should not be made an issue in this work.

The second, 'A Book of Multiplication Tables' is much more complicated to make. First of all it requires some care with the double-sided photocopying needed. Check that the crosses in the middle of the two sheets match (within 2 mm if possible), and also that the 94× table does back the 89× table (and NOT the 65×).

The sheet has then to be folded in half three times. Do not rush this stage. Press the folds down well with some hard object (ruler or pen). The dotted lines are only a guide, the important thing is to match up the edges of the paper each time, and see that the dotted lines are on the **inside** of the fold. The titled front cover should always be on the **outside** of the fold.

A staple through the last fold (the hinged edge) and three cuts along the outer edges with a guillotine will complete the process. It is nowhere near as difficult as it sounds!

An alternative cover for the booklet is provided. This will have to be cut and pasted onto the master copy before the photocopying is done.

All of this would only be worthwhile if the tables might be used in some other work.

These same tables are also available in a straight sheet format if wanted. They can be found in the 'Division' unit (in the *tr01* menu).

It must be emphasised that there are many more ways of doing multiplication than those given here, though most of them are merely variations on a general theme. The ones suggested in this unit are those considered to offer the most useful insights into the historic development of this particular piece of arithmetic so some might appreciate that

"No, it hasn't always been like that!"

Exactly how any or all of this material is used will be, as always, a matter for individual teachers to decide and plan for.

It can be surprising to see the lengths to which mathematicians and other professional users of mathematics have gone to over the centuries to simplify the multiplication process. But remember that even so great a mathematician as Leibnitz (1646 to 1716) who invented the calculus said

*"It is unworthy of excellent men to lose hours  
like slaves in the labour of calculation."*

(A feeling which many of our pupils share!)

# Notes on Multiplication Methods ~ 2

## Using Doubling and Halving

### 1. The Ancient Egyptian Method

This is probably the oldest of all the written methods. It is of some interest because of its links with the binary form of a number, and also because of the simplicity of the idea - it needs only an ability to be able to multiply by 2 and add.

However it should also be appreciated that in those days (1000 BC or so) such things were not for 'ordinary folk' who would have had no need of such a skill. It was very much the preserve of professionals such as astrologers, architects and mathematicians.

### 2. The Russian Peasant Method

This method was more widely used than its name might imply. It was in fact taught and practised throughout Europe for quite some time. It is a less demanding algorithm than the previous one since the numbers to be crossed off are easier to identify. It is interesting to write them side by side and see how they are doing the same thing. The link of course is the binary system underlying both.

<table style="border-collapse: collapse;"> <tr><td style="padding: 2px 10px;">1</td><td style="padding: 2px 10px;">268</td></tr> <tr><td style="padding: 2px 10px;">2</td><td style="padding: 2px 10px;">536</td></tr> <tr><td style="padding: 2px 10px;"><del>4</del></td><td style="padding: 2px 10px;"><del>1072</del></td></tr> <tr><td style="padding: 2px 10px;">8</td><td style="padding: 2px 10px;">2144</td></tr> <tr><td style="padding: 2px 10px;"><del>16</del></td><td style="padding: 2px 10px;"><del>4288</del></td></tr> <tr><td style="padding: 2px 10px;">32</td><td style="padding: 2px 10px;">8576</td></tr> <tr><td style="padding: 2px 10px;"><u>43</u></td><td style="padding: 2px 10px;"><u>11524</u></td></tr> </table>	1	268	2	536	<del>4</del>	<del>1072</del>	8	2144	<del>16</del>	<del>4288</del>	32	8576	<u>43</u>	<u>11524</u>	<table style="border-collapse: collapse;"> <tr><td style="padding: 2px 10px;">43</td><td style="padding: 2px 10px;">268</td></tr> <tr><td style="padding: 2px 10px;">21</td><td style="padding: 2px 10px;">536</td></tr> <tr><td style="padding: 2px 10px;"><del>10</del></td><td style="padding: 2px 10px;"><del>1072</del></td></tr> <tr><td style="padding: 2px 10px;">5</td><td style="padding: 2px 10px;">2144</td></tr> <tr><td style="padding: 2px 10px;"><del>2</del></td><td style="padding: 2px 10px;"><del>4288</del></td></tr> <tr><td style="padding: 2px 10px;">1</td><td style="padding: 2px 10px;">8576</td></tr> <tr><td style="padding: 2px 10px;"><u>1</u></td><td style="padding: 2px 10px;"><u>11524</u></td></tr> </table>	43	268	21	536	<del>10</del>	<del>1072</del>	5	2144	<del>2</del>	<del>4288</del>	1	8576	<u>1</u>	<u>11524</u>
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## Using the Gelosia Method

(Pronounced jee - low - see - uh)

Any analysis or explanation of why this method works is probably best linked to looking at the place values of the different figures involved.

The worked example from the sheet is shown on the right.

2	6	8	×
8	2	3	4
6	1	8	3

The lower diagram replaces each of the the figures in the top diagram with its place value, using the usual notation of H T U, with the addition of Th for thousands. The neat way each place value falls into line along the diagonals makes it very clear why the method works. It also makes it very apparent why zeros must be included in the numbers when necessary.

It is easy to extend the diagram to cover tens of thousands and upwards or, tenths, hundredths, thousandths and so on downwards, and thus show why placing the decimal point works as it does.

H	T	U	×
Th	H	T	T
H	T	U	U

This method almost certainly originated in India, as did so much of our arithmetic. From there it spread outwards to China, Persia and the Arab world. It reached Europe through Italy where it first appeared in the 14th century.

The name 'gelosia' was given to it in Italy since the necessary grid resembled the lattices or gratings (= gelosia) which were made in metal and fastened in front of the windows of the houses for security.

It has been claimed that Napier got the idea for his rods from this method. Whether or not he did, the possibility can clearly be seen. In fact, multiplication tables and a knowledge of place value could be dispensed with by using the two methods together. Once the grid has been drawn, a set of Napier's Rods would provide all the needed answers to complete it, and the final addition would be the only arithmetic necessary.

One practical point. Do not slow things up by drawing precisely ruled grids. It is quite sufficient to write down the two numbers (adequately spaced) and sketch the grid around them free-hand.

## Notes on Multiplication Methods ~ 3

### Using Quarter Squares

The reason why this method works can only be justified by the use of algebra.

It can be shown that

$$ab \equiv \frac{(a + b)^2}{4} - \frac{(a - b)^2}{4}$$

Given  $a$  and  $b$  are the two numbers to be multiplied together, and the two terms on the right are quarter squares (as defined on the sheet) then the algorithm follows from the above identity. And (provided the requisite tables are available) multiplication has been replaced by one addition and two subtractions without need for any multiplication facts to be known or found.

In passing it is of interest to consider (and prove) why the remainder can be disregarded.

Note that this method is completely accurate in giving all the figures possible in the final answer. Its big limitations are that

- (a) a set of tables is needed and,
- (b) the size of the numbers it can handle is constrained by the size of the tables (which has to cover the sum of the two numbers being multiplied).

The second limitation is not too bad. The sample sheet included here goes to 600, so it is easy to visualize the smallish book which would be needed to reach 100,000. That means an accuracy of 4 significant figures (or better) which should cover most practical needs. Approximations allow the tables to be used for larger numbers.

The method was used (but how widely is not known) and the necessary tables to enable this were published in the late 1800's.

### Using Logarithms

This is a much simplified version of what was once taught in the classrooms of every secondary school. The words '*characteristic*' and '*mantissa*' do not appear, and only numbers greater than 1 are dealt with to avoid that very troublesome negative characteristic.

Also only 3-figure tables are used and the customary difference table does not appear. However, the essentials required to give a glimpse of what was done are there. In modern terms this could be called "Logarithms Lite".

The characteristic which is so necessary to control the place value has been replaced here by the e-notation. This is the notation introduced by the IEEE (= Institute of Electrical and Electronic Engineers) and used in all computer work. Gaining an acquaintance with this notation might be regarded as a valuable by-product of this piece of work.

One of the inadequacies of 3-figure tables is very apparent once the numbers go beyond 5.0 when many of the logarithmic values are repeated. This cannot be avoided without moving on to 4 or 5-figure tables.

Sheet 1 does not require any knowledge of the characteristic as the examples are chosen so as not to go beyond the range of the table. Also there are no ambiguous values for the logarithms.

Sheet 2 introduces the idea of the e-notation.

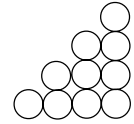
Some help might be needed with the business of using the tables, first in finding the logarithm from the number, and then the much more difficult matter of reversing that process.

Further work could be done with numbers less than 1 and division.

# Notes on Multiplication Methods ~ 4

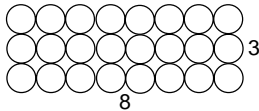
## Using Triangle Numbers

This method has the big advantage that it can be easily explained or justified diagrammatically, without any recourse to algebra, except for the generalisation at the end. For this purpose it is better to change the conventionally drawn triangle numbers (as equilateral triangles) into right-angled triangles. The appropriate drawing for  $T_4$  is shown on the right.

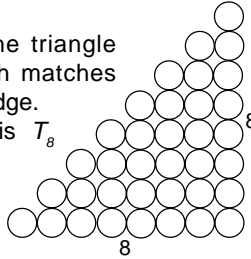


We will do the sum  $8 \times 3$  using five diagrams.

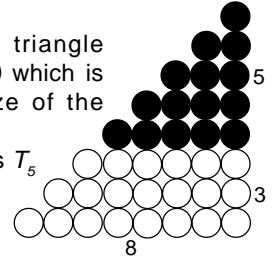
**1.** The sum  $8 \times 3$  requires the number of objects making this rectangle to be counted.



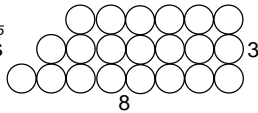
**2.** First make the triangle number which matches the longest edge. In this case it is  $T_8$ .



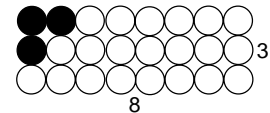
**3.** Cut away the triangle (shown in black) which is above the size of the rectangle. In this case it is  $T_5$ .



**4.** Triangle removed was  $T_5$  or  $T_{(8-3)}$  leaving this incomplete rectangle



**5.** Complete the rectangle with smallest possible triangle (shown in black). In this case  $T_2$  or  $T_{(3-1)}$



Physically we have shown that

$$\begin{aligned}
 8 \times 3 &= 36 - 15 + 3 \\
 &= T_8 - T_5 + T_2 \\
 &= T_8 - T_{(8-3)} + T_{(3-1)}
 \end{aligned}$$

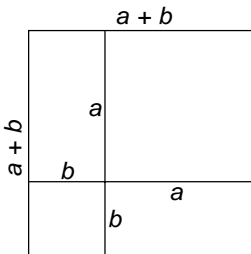
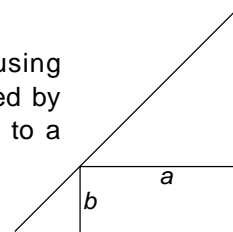
Which generalises to

$$a \times b = T_a - T_{(a-b)} + T_{(b-1)}$$

Rearranged for the algorithm

$$a \times b = T_a + T_{(b-1)} - T_{(a-b)}$$

**Another approach** to using triangle numbers is implied by this diagram which leads to a different algorithm.



**A similar thing** can be done using square numbers (or indeed any polygon numbers).

The identity

$$(a + b)^2 = a^2 + b^2 + 2ab$$

which is illustrated on the left can be rearranged to give  $ab =$

and the required algorithm can be produced from that. A table giving the values of square numbers  $S_n$  is available.

Though interesting, and illuminating, to see how these work they were never more than mathematical curiosities, and (unlike the method of quarter squares) there appears to be no evidence that they were ever used in any practical way. One advantage this method does have over quarter squares is that, in this case the largest number in the tables is also the largest possible multiplier.

# Multiplication using Doubling and Halving

## 1. The Ancient Egyptian Method

Consider the sum  $268 \times 43$

First write out two columns of numbers side by side.

The left-hand column starts with 1.

The right-hand column starts with the **larger** of the two numbers being multiplied (268).

The columns are formed by **doubling** each of the previous numbers, **stopping** only when the figure in the left-hand column would become **bigger** than (or equal to) the other number being multiplied (43).

The final result would be as shown on the right.

Take care to keep the columns lined up, both vertically and horizontally.

1	268
2	536
4	1072
8	2144
16	4288
32	8576

The next stage is to look at the left-hand column and find the numbers which are needed to add up to the value of the smaller multiplier (43).

In this case the necessary set is  $1 + 2 + 8 + 32 = 43$

*(Note that the bottom number should always be needed, and it is best working backwards from that, subtracting as you go.)*

Cross out all the numbers in that column not needed **together with** the numbers in the right-hand column which are on the same level.

Finally, add up the numbers in the right-hand column which have not been crossed out.

That is the answer. The completed sum is shown on the right.

So,  $268 \times 43 = 11524$

1	268
2	536
<del>4</del>	<del>1072</del>
8	2144
<del>16</del>	<del>4288</del>
32	8576
43	<b>11524</b>

**Try these**, using the Ancient Egyptian method.

$26 \times 314$      $41 \times 61$      $71 \times 213$      $55 \times 97$      $32 \times 104$      $251 \times 21$

## 2. The Russian Peasant Method

Consider the sum  $268 \times 43$

First write out two columns of numbers side by side.

Each column starts with one of the two numbers to be multiplied. (43 and 268)

Make one column by **doubling** each of the previous numbers.

Make the other column by **halving** each of the previous numbers, ignoring any remainders, and **stopping** at 1.

*(It is best to make the halving column under the smaller of the two multipliers)*

The final result would be as shown on the right.

Take care to keep the columns lined up, both vertically and horizontally.

43	268
21	536
10	1072
5	2144
2	4288
1	8576

The next stage is to look at the **halving** column and find all the **even** numbers.

In this case they are 10 and 2

Cross out the even numbers in that column **together with** the numbers in the right-hand column which are on the same level.

Finally, add up the numbers in the right-hand column which have not been crossed out.

That is the answer. The completed sum is shown on the right.

So,  $268 \times 43 = 11524$

43	268
21	536
<del>10</del>	<del>1072</del>
5	2144
<del>2</del>	<del>4288</del>
1	8576
	<b>11524</b>

**Try these**, using the Russian Peasant method.

$24 \times 316$      $30 \times 54$      $63 \times 81$      $128 \times 231$      $71 \times 483$      $327 \times 45$

## Multiplication using the Gelosia Method

Consider the sum  $268 \times 43$  (That is 3 digits  $\times$  2 digits)

Sketch a grid like that on the right, and place the two numbers as shown.

2	6	8	×
/	/	/	4
/	/	/	3

Next, do the six single-digit multiplications formed by matching the separate numbers across the top with those on the right-hand edge.

In this case, it means doing the six sums shown on the right

$2 \times 4$	$6 \times 4$	$8 \times 4$
$2 \times 3$	$6 \times 3$	$8 \times 3$

and the answers to those multiplications are written in the appropriate boxes of the grid for each number-pair.

2	6	8	×
8	2	3	4
6	1	8	2
/	/	/	3

No answer can have more than two digits. (tens and units)

Any **tens** (L.H. digit) are written **above** the diagonal line,

The **units** (R.H. digit) are written **below** the diagonal line. (It may be zero)

The completed grid should look like that on the right.

The final answer is produced by adding (the results of the multiplication only) along the diagonals, working from right to left, and carrying figures as necessary, and this is shown on the right.

2	6	8	×
8	2	3	4
6	1	8	2
/	/	/	3

1    1
5    2    4

So,  $268 \times 43 = 11524$

**Try these**, using the gelosia method.

$386 \times 24$

$758 \times 34$

$76 \times 89$

$402 \times 31$

$999 \times 78$

$478 \times 219$

**Decimal points** are dealt with very simply.

Consider the sum  $2.68 \times 4.3$

Work exactly as before with  $268 \times 43$  and then put the two decimal points in those numbers on the vertical and horizontal lines respectively.

Follow those two lines into the grid until they meet, and then, moving down the diagonal line from there, put the decimal point in the answer.

All of this can be seen in the diagram on the right by following the arrows.

2	6	8	×
8	2	3	4
6	1	8	2
/	/	/	3

1    1    .
5    2    4

So,  $2.68 \times 4.3 = 11.524$

**Try these**, using the gelosia method.

First, by using the grids already created in the previous work, write down the answers to these:

$3.86 \times 24$

$75.8 \times 3.4$

$7.6 \times 89$

$40.2 \times 31$

$9.99 \times 78$

$47.8 \times 2.19$

and then do these:

$12.3 \times 4.6$

$7.32 \times 18$

$508 \times 3.7$

$0.617 \times 3.8$

$0.059 \times 23$

$0.074 \times 0.038$

## Multiplication using Quarter Squares

The sheet headed **Values of Quarter Squares  $Q_n$**  is needed

The quarter square of a number, is the value of that number multiplied by itself and then divided by 4, ignoring any remainder (if there is one).

*Examples*

The quarter square of 2 is:  $2 \times 2 \div 4 = 4 \div 4 = 1$

3 is:  $3 \times 3 \div 4 = 9 \div 4 = 2 \text{ rem } 1$  which becomes 2

5 is:  $5 \times 5 \div 4 = 25 \div 4 = 6 \text{ rem } 1$  which becomes 6  
and so on.

Quarter squares would be of little use for the purpose of simplifying multiplication if they had to be worked out each time they were needed and so a table is used to look up their values.

Look at the sheet: Values of Quarter Squares and confirm that the quarter square of 413 is 42 642 (or 42,642 if preferred).

Having such a table enables us to change a multiplication sum into a much simpler matter of addition and subtraction.

Consider the sum  $268 \times 43$

First add them:  $268 + 43 = 311$  and find the quarter square of that = 24 180

Then subtract them:  $268 - 43 = 225$  and find the quarter square of that = 12 656

Finally, subtract the quarter squares to get the final answer = 11 524  
and  $268 \times 43 = \mathbf{11\ 524}$

It could be set out something like this:

	268		
×	43		$Q_n$
+	311		24 180
−	225		12 656 −
			<b>11 524</b>

**Try these**, using the quarter squares method.

$245 \times 32$

$286 \times 45$

$364 \times 123$

$74 \times 36$

$376 \times 21$

$443 \times 157$




## Multiplication using Triangle Numbers

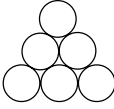
The sheet headed **Values of Triangle Numbers  $T_n$**  is needed



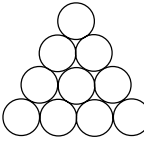
$T_1 = 1$



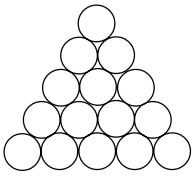
$T_2 = 3$



$T_3 = 6$



$T_4 = 10$



$T_5 = 15$

Triangle numbers are those that can be made in the way shown above where the first five are drawn.

The actual value of any triangle number can be worked out using

$$T_n = n \times (n + 1) \div 2$$

*Example*

The value of the triangle number for 8 is:  $8 \times (8 + 1) \div 2 = 8 \times 9 \div 2 = 36$

Triangle Numbers would be of little use for the purpose of simplifying multiplication if they had to be worked out each time they were needed and so a table is used to look up their values.

Look at the sheet: Values of Triangle Numbers and confirm that the triangle number corresponding to 413 is 85 491 (or 85,491 if preferred).

Having such a table enables us to change a multiplication sum into a much simpler matter of addition and subtraction.

Consider the sum  $268 \times 43$

Find the triangle number corresponding to the larger (268) = 36 046

Subtract 1 from the smaller ( $43 - 1 = 42$ ) and find its triangle number = 12 656

Add those two triangle numbers = 36 949

Subtract the two multiplying numbers ( $268 - 43 = 225$ )  
and find the triangle number for that = 25 425

Subtract that from the previous result to get the final answer = 11 524  
and  $268 \times 43 = 11\ 524$

It could be set out something like this:

		$T_n$
	268	36 046
	$43 - 1 = 42$	903 +
		36 949
-	225	25 425 -
		<b>11 524</b>

**Try these**, using the triangle numbers method.

$213 \times 87$

$367 \times 42$

$179 \times 53$

$485 \times 132$

$593 \times 341$

$259 \times 34$

# Multiplication using Logarithms ~ 1

The sheet headed **Values of Logarithms  $L_n$**  is needed

We will not concern ourselves here with what logarithms are, but only how to use them for multiplication. (They can also be used for division.)

First of all, every number has a logarithm associated with it and that association is unique. In other words, given any number it has only one logarithm and, just as importantly, given any logarithm it can only represent one number.

We will start with a very simple example of how we use logarithms.

Consider the sum  $2 \times 4$

To use the table of logarithms we need to see this as  $2.00 \times 4.00$

From the table we can find that 2.00 has a logarithm of 0.301

and that 4.00 has a logarithm of 0.602

Then we ADD the two logarithms to get 0.903

That is the answer to the sum but, it is a logarithm, so we need to find what number it represents.

Using the table (in reverse) shows that 0.903 is the logarithm of 8.00

So  $2.00 \times 4.00 = 8.00$  (Hardly a surprise, but good to see it works.)

Now consider the much harder sum  $2.15 \times 3.62$

We find their logarithms are 0.332 and 0.559 and these add to 0.891

The logarithm 0.891 matches the number 7.78

So  $2.15 \times 3.62 = 7.78$

It could be set out something like this:

$N$	$L_n$
2.15	0.332
$\times$ 3.62	0.559 +
<b>7.78</b>	<b>0.891</b>

The full answer should have been 7.783 but it must be recognised that the very simplified table we are using is not capable of that sort of accuracy. We wish only to establish how logarithms are used and, for most practical purposes, this accuracy is adequate.

**Try doing these, using logarithms**

$2 \times 3$

$3 \times 3$

$1.92 \times 3.36$

$5.18 \times 1.45$

$3.55 \times 1.29$

$4.07 \times 1.93$

## Multiplication using Logarithms ~ 2

Notice that the table of logarithms covers only numbers from 1.00 to 9.99 and that is true for all tables of logarithms. For greater accuracy it might be 1.0000 to 9.9999 but the principle is the same. So now we consider how to adapt all numbers to fit the tables and see how powerful this tool is in handling all multiplications.

For this we need to be familiar with a particular way of writing numbers which first sets them down as a value between 1 and 10 and then puts enough  $\times 10$ 's after them so they would be restored to their correct value.

For example 43 can be written as  $4.3 \times 10$   
268 can be written as  $2.68 \times 10 \times 10$

There is a special notation used for this which writes the above numbers as

4.3e1 and 2.68e2 respectively

where the number after the 'e' shows how many 10's are needed.

*Or it can be thought of as the amount that the decimal point must be moved to the **right** to restore the correct value.*

Now when we look up the logarithms of 4.3 and 2.68 we replace the 0 on the front with the e-number (if there is one).

So that the logarithm of 43 ( $\log 4.3 = 0.633$  with  $e = 1$ ) becomes **1.633**

and the logarithm of 268 ( $\log 2.68 = 0.428$  with  $e = 2$ ) becomes **2.428**

Then we add the two logarithms to get **4.061**

Remembering this stands for a logarithm of 0.061 with  $e = 4$

We next find 0.061 in the logarithm tables and see that it corresponds to the number 1.15

With  $e = 4$  we must multiply by 10, 4 times

$$1.15 \times 10 \times 10 \times 10 \times 10 = 11500$$

$$\text{More accurately } 43 \times 268 = 11524$$

But we can write  $43 \times 268 = 11500$  (to 3 significant figures)

It could be set out something like this:

$N$	$L_n$
43	1.633
$\times$ 268	2.428 +
<b>1.15</b>	<b>4.061</b>

**Try doing these**, using logarithms

$37 \times 26$

$45 \times 78$

$274 \times 63$

$517 \times 842$

$65.4 \times 173$

$3610 \times 904$

Values of Quarter Squares  $Q_n$  for  $n = 1$  to 600

1	0
2	1
3	2
4	4
5	6
6	9
7	12
8	16
9	20
10	25
11	30
12	36
13	42
14	49
15	56
16	64
17	72
18	81
19	90
20	100
21	110
22	121
23	132
24	144
25	156
26	169
27	182
28	196
29	210
30	225
31	240
32	256
33	272
34	289
35	306
36	324
37	342
38	361
39	380
40	400
41	420
42	441
43	462
44	484
45	506
46	529
47	552
48	576
49	600
50	625
51	650
52	676
53	702
54	729
55	756
56	784
57	812
58	841
59	870
60	900
61	930
62	961
63	992
64	1024
65	1056
66	1089
67	1122
68	1156
69	1190
70	1225
71	1260
72	1296
73	1332
74	1369
75	1406
76	1444
77	1482
78	1521
79	1560
80	1600
81	1640
82	1681
83	1722
84	1764
85	1806
86	1849
87	1892
88	1936
89	1980
90	2025
91	2070
92	2116
93	2162
94	2209
95	2256
96	2304
97	2352
98	2401
99	2450
100	2500
101	2550
102	2601
103	2652
104	2704
105	2756
106	2809
107	2862
108	2916
109	2970
110	3025
111	3080
112	3136
113	3192
114	3249
115	3306
116	3364
117	3421
118	3481
119	3540
120	3600
121	3660
122	3721
123	3782
124	3844
125	3906
126	3969
127	4032
128	4096
129	4160
130	4225
131	4290
132	4356
133	4422
134	4489
135	4556
136	4624
137	4692
138	4761
139	4830
140	4900
141	4970
142	5041
143	5112
144	5184
145	5256
146	5329
147	5402
148	5476
149	5550
150	5625
151	5700
152	5776
153	5852
154	5929
155	6006
156	6084
157	6162
158	6241
159	6320
160	6400
161	6480
162	6561
163	6642
164	6724
165	6806
166	6889
167	6972
168	7056
169	7140
170	7225
171	7310
172	7396
173	7482
174	7569
175	7656
176	7744
177	7832
178	7921
179	8010
180	8100
181	8190
182	8281
183	8372
184	8464
185	8556
186	8649
187	8742
188	8836
189	8930
190	9025
191	9120
192	9216
193	9312
194	9409
195	9506
196	9604
197	9702
198	9801
199	9900
200	10 000
201	10 100
202	10 201
203	10 302
204	10 404
205	10 506
206	10 609
207	10 712
208	10 816
209	10 920
210	11 025
211	11 130
212	11 236
213	11 342
214	11 449
215	11 556
216	11 664
217	11 772
218	11 881
219	11 990
220	12 100
221	12 210
222	12 321
223	12 432
224	12 544
225	12 656
226	12 769
227	12 882
228	12 996
229	13 110
230	13 225
231	13 340
232	13 456
233	13 572
234	13 689
235	13 806
236	13 924
237	14 042
238	14 161
239	14 280
240	14 400
241	14 520
242	14 641
243	14 762
244	14 884
245	15 006
246	15 129
247	15 252
248	15 376
249	15 500
250	15 625
251	15 750
252	15 876
253	16 002
254	16 129
255	16 256
256	16 384
257	16 512
258	16 641
259	16 770
260	16 900
261	17 030
262	17 161
263	17 292
264	17 424
265	17 556
266	17 689
267	17 822
268	17 956
269	18 090
270	18 225
271	18 360
272	18 496
273	18 632
274	18 769
275	18 906
276	19 044
277	19 182
278	19 321
279	19 460
280	19 600
281	19 740
282	19 881
283	20 022
284	20 164
285	20 306
286	20 449
287	20 592
288	20 736
289	20 880
290	21 025
291	21 170
292	21 316
293	21 462
294	21 609
295	21 756
296	21 904
297	22 052
298	22 201
299	22 350
300	22 500
301	22 650
302	22 801
303	22 952
304	23 104
305	23 256
306	23 409
307	23 562
308	23 716
309	23 870
310	24 025
311	24 180
312	24 336
313	24 492
314	24 649
315	24 806
316	24 964
317	25 122
318	25 281
319	25 440
320	25 600
321	25 760
322	25 921
323	26 082
324	26 244
325	26 406
326	26 569
327	26 732
328	26 896
329	27 060
330	27 225
331	27 390
332	27 556
333	27 722
334	27 889
335	28 056
336	28 224
337	28 392
338	28 561
339	28 730
340	28 900
341	29 070
342	29 241
343	29 412
344	29 584
345	29 756
346	29 929
347	30 102
348	30 276
349	30 450
350	30 625
351	30 800
352	30 976
353	31 152
354	31 329
355	31 506
356	31 684
357	31 862
358	32 041
359	32 220
360	32 400
361	32 580
362	32 761
363	32 942
364	33 124
365	33 306
366	33 489
367	33 672
368	33 856
369	34 040
370	34 225
371	34 410
372	34 596
373	34 782
374	34 969
375	35 156
376	35 344
377	35 532
378	35 721
379	35 910
380	36 100
381	36 290
382	36 481
383	36 672
384	36 864
385	37 056
386	37 249
387	37 442
388	37 636
389	37 830
390	38 025
391	38 220
392	38 416
393	38 612
394	38 809
395	39 006
396	39 204
397	39 402
398	39 601
399	39 800
400	40 000
401	40 200
402	40 401
403	40 602
404	40 804
405	41 006
406	41 209
407	41 412
408	41 616
409	41 820
410	42 025
411	42 230
412	42 436
413	42 642
414	42 849
415	43 056
416	43 264
417	43 472
418	43 681
419	43 890
420	44 100
421	44 310
422	44 521
423	44 732
424	44 944
425	45 156
426	45 369
427	45 582
428	45 796
429	46 010
430	46 225
431	46 440
432	46 656
433	46 872
434	47 089
435	47 306
436	47 524
437	47 742
438	47 961
439	48 180
440	48 400
441	48 620
442	48 841
443	49 062
444	49 284
445	49 506
446	49 729
447	49 952
448	50 176
449	50 400
450	50 625
451	50 850
452	51 076
453	51 302
454	51 529
455	51 756
456	51 984
457	52 212
458	52 441
459	52 670
460	52 900
461	53 130
462	53 361
463	53 592
464	53 824
465	54 056
466	54 289
467	54 522
468	54 756
469	54 990
470	55 225
471	55 460
472	55 696
473	55 932
474	56 169
475	56 406
476	56 644
477	56 882
478	57 121
479	57 360
480	57 600
481	57 840
482	58 081
483	58 322
484	58 564
485	58 806
486	59 049
487	59 292
488	59 536
489	59 780
490	60 025
491	60 270
492	60 516
493	60 762
494	61 009
495	61 256
496	61 504
497	61 752
498	62 001
499	62 250
500	62 500
501	62 750
502	63 001
503	63 252
504	63 504
505	63 756
506	64 009
507	64 262
508	64 516
509	64 770
510	65 025
511	65 280
512	65 536
513	65 792
514	66 049
515	66 306
516	66 564
517	66 822
518	67 081
519	67 340
520	67 600
521	67 860
522	68 121
523	68 382
524	68 644
525	68 906
526	69 169
527	69 432
528	69 696
529	69 960
530	70 225
531	70 490
532	70 756
533	71 022
534	71 289
535	71 556
536	71 824
537	



Values of Square Numbers  $S_n$  for  $n = 1$  to 600

1	1	1
2	4	4
3	9	9
4	16	16
5	25	25
6	36	36
7	49	49
8	64	64
9	81	81
10	100	100
11	121	121
12	144	144
13	169	169
14	196	196
15	225	225
16	256	256
17	289	289
18	324	324
19	361	361
20	400	400
21	441	441
22	484	484
23	529	529
24	576	576
25	625	625
26	676	676
27	729	729
28	784	784
29	841	841
30	900	900
31	961	961
32	1024	1024
33	1089	1089
34	1156	1156
35	1225	1225
36	1296	1296
37	1369	1369
38	1444	1444
39	1521	1521
40	1600	1600
41	1681	1681
42	1764	1764
43	1849	1849
44	1936	1936
45	2025	2025
46	2116	2116
47	2209	2209
48	2304	2304
49	2401	2401
50	2500	2500
51	2601	2601
52	2704	2704
53	2809	2809
54	2916	2916
55	3025	3025
56	3136	3136
57	3249	3249
58	3364	3364
59	3481	3481
60	3600	3600
61	3721	3721
62	3844	3844
63	3969	3969
64	4096	4096
65	4225	4225
66	4356	4356
67	4489	4489
68	4624	4624
69	4761	4761
70	4900	4900
71	5041	5041
72	5184	5184
73	5329	5329
74	5476	5476
75	5625	5625
76	5776	5776
77	5929	5929
78	6084	6084
79	6241	6241
80	6400	6400
81	6561	6561
82	6724	6724
83	6889	6889
84	7056	7056
85	7225	7225
86	7396	7396
87	7569	7569
88	7744	7744
89	7921	7921
90	8100	8100
91	8281	8281
92	8464	8464
93	8649	8649
94	8836	8836
95	9025	9025
96	9216	9216
97	9409	9409
98	9604	9604
99	9801	9801
100	10000	10000
101	10201	10201
102	10404	10404
103	10609	10609
104	10816	10816
105	11025	11025
106	11236	11236
107	11449	11449
108	11664	11664
109	11881	11881
110	12100	12100
111	12321	12321
112	12544	12544
113	12769	12769
114	12996	12996
115	13225	13225
116	13456	13456
117	13689	13689
118	13924	13924
119	14161	14161
120	14400	14400
121	14641	14641
122	14884	14884
123	15129	15129
124	15376	15376
125	15625	15625
126	15876	15876
127	16129	16129
128	16384	16384
129	16641	16641
130	16900	16900
131	17161	17161
132	17424	17424
133	17689	17689
134	17956	17956
135	18225	18225
136	18496	18496
137	18769	18769
138	19044	19044
139	19321	19321
140	19600	19600
141	19881	19881
142	20164	20164
143	20449	20449
144	20736	20736
145	21025	21025
146	21316	21316
147	21609	21609
148	21904	21904
149	22201	22201
150	22500	22500
151	22801	22801
152	23104	23104
153	23409	23409
154	23716	23716
155	24025	24025
156	24336	24336
157	24649	24649
158	24964	24964
159	25281	25281
160	25600	25600
161	25921	25921
162	26244	26244
163	26569	26569
164	26896	26896
165	27225	27225
166	27556	27556
167	27889	27889
168	28224	28224
169	28561	28561
170	28900	28900
171	29241	29241
172	29584	29584
173	29929	29929
174	30276	30276
175	30625	30625
176	30976	30976
177	31329	31329
178	31684	31684
179	32041	32041
180	32400	32400
181	32761	32761
182	33124	33124
183	33489	33489
184	33856	33856
185	34225	34225
186	34596	34596
187	34969	34969
188	35344	35344
189	35721	35721
190	36100	36100
191	36481	36481
192	36864	36864
193	37249	37249
194	37636	37636
195	38025	38025
196	38416	38416
197	38809	38809
198	39204	39204
199	39601	39601
200	40000	40000
201	40401	40401
202	40804	40804
203	41209	41209
204	41616	41616
205	42025	42025
206	42436	42436
207	42849	42849
208	43264	43264
209	43681	43681
210	44100	44100
211	44521	44521
212	44944	44944
213	45369	45369
214	45796	45796
215	46225	46225
216	46656	46656
217	47089	47089
218	47524	47524
219	47961	47961
220	48400	48400
221	48841	48841
222	49284	49284
223	49729	49729
224	50176	50176
225	50625	50625
226	51076	51076
227	51529	51529
228	51984	51984
229	52441	52441
230	52900	52900
231	53361	53361
232	53824	53824
233	54289	54289
234	54756	54756
235	55225	55225
236	55696	55696
237	56169	56169
238	56644	56644
239	57121	57121
240	57600	57600
241	58081	58081
242	58564	58564
243	59049	59049
244	59536	59536
245	60025	60025
246	60516	60516
247	61009	61009
248	61504	61504
249	62001	62001
250	62500	62500
251	63001	63001
252	63504	63504
253	64009	64009
254	64516	64516
255	65025	65025
256	65536	65536
257	66049	66049
258	66564	66564
259	67081	67081
260	67600	67600
261	68121	68121
262	68644	68644
263	69169	69169
264	69696	69696
265	70225	70225
266	70756	70756
267	71289	71289
268	71824	71824
269	72361	72361
270	72900	72900
271	73441	73441
272	73984	73984
273	74529	74529
274	75076	75076
275	75625	75625
276	76176	76176
277	76729	76729
278	77284	77284
279	77841	77841
280	78400	78400
281	78961	78961
282	79524	79524
283	80089	80089
284	80656	80656
285	81225	81225
286	81796	81796
287	82369	82369
288	82944	82944
289	83521	83521
290	84100	84100
291	84681	84681
292	85264	85264
293	85849	85849
294	86436	86436
295	87025	87025
296	87616	87616
297	88209	88209
298	88804	88804
299	89401	89401
300	90000	90000
301	90601	90601
302	91204	91204
303	91809	91809
304	92416	92416
305	93025	93025
306	93636	93636
307	94249	94249
308	94864	94864
309	95481	95481
310	96100	96100
311	96721	96721
312	97344	97344
313	97969	97969
314	98596	98596
315	99225	99225
316	99856	99856
317	100489	100489
318	101124	101124
319	101761	101761
320	102400	102400
321	103041	103041
322	103684	103684
323	104329	104329
324	104976	104976
325	105625	105625
326	106276	106276
327	106929	106929
328	107584	107584
329	108241	108241
330	108900	108900
331	109561	109561
332	110224	110224
333	110889	110889
334	111556	111556
335	112225	112225
336	112896	112896
337	113569	113569
338	114244	114244
339	114921	114921
340	115600	115600
341	116281	116281
342	116964	116964
343	117649	117649
344	118336	118336
345	119025	119025
346	119716	119716
347	120409	120409
348	121104	121104
349	121801	121801
350	122500	122500
351	123201	123201
352	123904	123904
353	124609	124609
354	125316	125316
355	126025	126025
356	126736	126736
357	127449	127449
358	128164	128164
359	128881	128881
360	129600	129600
361	130321	130321
362	131044	131044
363	131769	131769
364	132496	132496
365	133225	133225
366	133956	133956
367	134689	134689
368	135424	135424
369	136161	136161
370	136900	136900
371	137641	137641
372	138384	138384
373	139129	139129
374	139876	139876
375	140625	140625
376	141376	141376
377	142129	142129
378	142884	142884
379	143641	143641
380	144400	144400
38		

# Values of Logarithms $L_n$ for $n = 1.0$ to $9.99$

1.0	0.000	0.004	0.009	0.013	0.017	0.021	0.025	0.029	0.033	0.037	0.041
1.1	0.041	0.045	0.049	0.053	0.057	0.061	0.064	0.068	0.072	0.076	0.079
1.2	0.079	0.083	0.086	0.090	0.093	0.097	0.100	0.104	0.107	0.111	0.114
1.3	0.114	0.117	0.121	0.124	0.127	0.130	0.134	0.137	0.140	0.143	0.146
1.4	0.146	0.149	0.152	0.155	0.158	0.161	0.164	0.167	0.170	0.173	0.176
1.5	0.176	0.179	0.182	0.185	0.188	0.190	0.193	0.196	0.199	0.201	0.204
1.6	0.204	0.207	0.210	0.212	0.215	0.217	0.220	0.222	0.225	0.228	0.230
1.7	0.230	0.233	0.236	0.238	0.241	0.243	0.245	0.248	0.250	0.253	0.255
1.8	0.255	0.258	0.260	0.262	0.265	0.267	0.270	0.272	0.274	0.276	0.279
1.9	0.279	0.281	0.283	0.286	0.288	0.290	0.292	0.294	0.297	0.299	0.301
2.0	0.301	0.303	0.305	0.308	0.310	0.312	0.314	0.316	0.318	0.320	0.322
2.1	0.322	0.324	0.326	0.328	0.330	0.332	0.334	0.336	0.338	0.340	0.342
2.2	0.342	0.344	0.346	0.348	0.350	0.352	0.354	0.356	0.358	0.360	0.362
2.3	0.362	0.364	0.365	0.367	0.369	0.371	0.373	0.375	0.377	0.378	0.380
2.4	0.380	0.382	0.384	0.386	0.387	0.389	0.391	0.393	0.394	0.396	0.398
2.5	0.398	0.400	0.401	0.403	0.405	0.407	0.408	0.410	0.412	0.413	0.415
2.6	0.415	0.417	0.418	0.420	0.422	0.423	0.425	0.427	0.428	0.430	0.431
2.7	0.431	0.433	0.435	0.436	0.438	0.439	0.441	0.442	0.444	0.446	0.447
2.8	0.447	0.449	0.450	0.452	0.453	0.455	0.456	0.458	0.459	0.461	0.462
2.9	0.462	0.464	0.465	0.467	0.468	0.470	0.471	0.473	0.474	0.476	0.477
3.0	0.477	0.479	0.480	0.481	0.483	0.484	0.486	0.487	0.489	0.490	0.491
3.1	0.491	0.493	0.494	0.496	0.497	0.498	0.500	0.501	0.502	0.504	0.505
3.2	0.505	0.507	0.508	0.509	0.510	0.512	0.513	0.515	0.516	0.517	0.519
3.3	0.519	0.520	0.521	0.522	0.524	0.525	0.526	0.528	0.529	0.530	0.531
3.4	0.531	0.533	0.534	0.535	0.537	0.538	0.539	0.540	0.542	0.543	0.544
3.5	0.544	0.545	0.547	0.548	0.549	0.550	0.551	0.553	0.554	0.555	0.556
3.6	0.556	0.558	0.559	0.560	0.561	0.562	0.563	0.565	0.566	0.567	0.568
3.7	0.568	0.569	0.571	0.572	0.573	0.574	0.575	0.577	0.579	0.580	0.581
3.8	0.580	0.581	0.582	0.583	0.584	0.585	0.587	0.588	0.589	0.590	0.591
3.9	0.591	0.592	0.593	0.594	0.596	0.597	0.598	0.599	0.600	0.601	0.602
4.0	0.602	0.603	0.604	0.605	0.606	0.607	0.609	0.610	0.611	0.612	0.613
4.1	0.613	0.614	0.615	0.616	0.617	0.618	0.619	0.620	0.621	0.622	0.623
4.2	0.623	0.624	0.625	0.626	0.627	0.628	0.629	0.630	0.631	0.632	0.633
4.3	0.633	0.634	0.635	0.636	0.637	0.638	0.639	0.640	0.641	0.642	0.643
4.4	0.643	0.644	0.645	0.646	0.647	0.648	0.649	0.650	0.651	0.652	0.653
4.5	0.653	0.654	0.655	0.656	0.657	0.658	0.659	0.660	0.661	0.662	0.663
4.6	0.663	0.664	0.665	0.666	0.667	0.667	0.668	0.669	0.670	0.671	0.672
4.7	0.672	0.673	0.674	0.675	0.676	0.677	0.678	0.679	0.679	0.680	0.681
4.8	0.681	0.682	0.683	0.684	0.685	0.686	0.687	0.688	0.688	0.689	0.690
4.9	0.690	0.691	0.692	0.693	0.694	0.695	0.695	0.696	0.697	0.698	0.699

To find the value of the logarithm of  $n$

Match the first two digits of  $n$  (containing the decimal point) with those in the left-hand column of the table. Go along that line of the table to the three figure decimal number which lies directly under the value at the head of the table matching the third digit of  $n$ .

That will be the logarithm of  $n$ .

5.0	0.699	0.700	0.702	0.703	0.704	0.705	0.706	0.707	0.708	0.709	0.710	0.711	0.712	0.713	0.714	0.715	0.716	0.717	0.718	0.719	0.720	0.721	0.722	0.723	0.724	0.725	0.726	0.727	0.728	0.729	0.730	0.731	0.732	0.733	0.734	0.735	0.736	0.737	0.738	0.739	0.740	0.741	0.742	0.743	0.744	0.745	0.746	0.747	0.748	0.749	0.750	0.751	0.752	0.753	0.754	0.755	0.756	0.757	0.758	0.759	0.760	0.761	0.762	0.763	0.764	0.765	0.766	0.767	0.768	0.769	0.770	0.771	0.772	0.773	0.774	0.775	0.776	0.777	0.778	0.779	0.780	0.781	0.782	0.783	0.784	0.785	0.786	0.787	0.788	0.789	0.790	0.791	0.792	0.793	0.794	0.795	0.796	0.797	0.798	0.799	0.800	0.801	0.802	0.803	0.804	0.805	0.806	0.807	0.808	0.809	0.810	0.811	0.812	0.813	0.814	0.815	0.816	0.817	0.818	0.819	0.820	0.821	0.822	0.823	0.824	0.825	0.826	0.827	0.828	0.829	0.830	0.831	0.832	0.833	0.834	0.835	0.836	0.837	0.838	0.839	0.840	0.841	0.842	0.843	0.844	0.845	0.846	0.847	0.848	0.849	0.850	0.851	0.852	0.853	0.854	0.855	0.856	0.857	0.858	0.859	0.860	0.861	0.862	0.863	0.864	0.865	0.866	0.867	0.868	0.869	0.870	0.871	0.872	0.873	0.874	0.875	0.876	0.877	0.878	0.879	0.880	0.881	0.882	0.883	0.884	0.885	0.886	0.887	0.888	0.889	0.890	0.891	0.892	0.893	0.894	0.895	0.896	0.897	0.898	0.899	0.900	0.901	0.902	0.903	0.904	0.905	0.906	0.907	0.908	0.909	0.910	0.911	0.912	0.913	0.914	0.915	0.916	0.917	0.918	0.919	0.920	0.921	0.922	0.923	0.924	0.925	0.926	0.927	0.928	0.929	0.930	0.931	0.932	0.933	0.934	0.935	0.936	0.937	0.938	0.939	0.940	0.941	0.942	0.943	0.944	0.945	0.946	0.947	0.948	0.949	0.950	0.951	0.952	0.953	0.954	0.955	0.956	0.957	0.958	0.959	0.960	0.961	0.962	0.963	0.964	0.965	0.966	0.967	0.968	0.969	0.970	0.971	0.972	0.973	0.974	0.975	0.976	0.977	0.978	0.979	0.980	0.981	0.982	0.983	0.984	0.985	0.986	0.987	0.988	0.989	0.990	0.991	0.992	0.993	0.994	0.995	0.996	0.997	0.998	0.999	1.000
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x	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

x	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

x	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

x	1	2	3	4	5	6	7	8	9
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2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
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5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

x	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

x	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

x	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

x	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81



**70**  $2 \times 2 = 4$   
 $3 \times 3 = 9$   
 $4 \times 4 = 16$   
 $5 \times 5 = 25$   
 $6 \times 6 = 36$   
 $7 \times 7 = 49$   
 $8 \times 8 = 64$   
 $9 \times 9 = 81$

**71**  $2 \times 3 = 6$   
 $3 \times 4 = 12$   
 $4 \times 5 = 20$   
 $5 \times 6 = 30$   
 $6 \times 7 = 42$   
 $7 \times 8 = 56$   
 $8 \times 9 = 72$

**72**  $2 \times 4 = 8$   
 $3 \times 5 = 15$   
 $4 \times 6 = 24$   
 $5 \times 7 = 35$   
 $6 \times 8 = 48$   
 $7 \times 9 = 63$

**73**  $3 \times 3 = 9$   
 $3 \times 4 = 12$   
 $3 \times 5 = 15$   
 $3 \times 6 = 18$   
 $3 \times 7 = 21$   
 $3 \times 8 = 24$   
 $3 \times 9 = 27$

**74**  $4 \times 4 = 16$   
 $4 \times 5 = 20$   
 $4 \times 6 = 24$   
 $4 \times 7 = 28$   
 $4 \times 8 = 32$   
 $4 \times 9 = 36$

**75**  $5 \times 5 = 25$   
 $5 \times 6 = 30$   
 $5 \times 7 = 35$   
 $5 \times 8 = 40$   
 $5 \times 9 = 45$

**76**  $6 \times 6 = 36$   
 $6 \times 7 = 42$   
 $6 \times 8 = 48$   
 $6 \times 9 = 54$

**77**  $7 \times 7 = 49$   
 $7 \times 8 = 56$   
 $7 \times 9 = 63$

**78**  $8 \times 8 = 64$   
 $8 \times 9 = 72$

**79**  $9 \times 9 = 81$

**53**  $3 \times 2 = 6$   
 $3 \times 3 = 9$   
 $3 \times 4 = 12$   
 $3 \times 5 = 15$   
 $3 \times 6 = 18$   
 $3 \times 7 = 21$   
 $3 \times 8 = 24$   
 $3 \times 9 = 27$

**54**  $4 \times 2 = 8$   
 $4 \times 3 = 12$   
 $4 \times 4 = 16$   
 $4 \times 5 = 20$   
 $4 \times 6 = 24$   
 $4 \times 7 = 28$   
 $4 \times 8 = 32$   
 $4 \times 9 = 36$

**55**  $5 \times 2 = 10$   
 $5 \times 3 = 15$   
 $5 \times 4 = 20$   
 $5 \times 5 = 25$   
 $5 \times 6 = 30$   
 $5 \times 7 = 35$   
 $5 \times 8 = 40$   
 $5 \times 9 = 45$

**56**  $6 \times 2 = 12$   
 $6 \times 3 = 18$   
 $6 \times 4 = 24$   
 $6 \times 5 = 30$   
 $6 \times 6 = 36$   
 $6 \times 7 = 42$   
 $6 \times 8 = 48$   
 $6 \times 9 = 54$

**57**  $7 \times 2 = 14$   
 $7 \times 3 = 21$   
 $7 \times 4 = 28$   
 $7 \times 5 = 35$   
 $7 \times 6 = 42$   
 $7 \times 7 = 49$   
 $7 \times 8 = 56$   
 $7 \times 9 = 63$

**58**  $8 \times 2 = 16$   
 $8 \times 3 = 24$   
 $8 \times 4 = 32$   
 $8 \times 5 = 40$   
 $8 \times 6 = 48$   
 $8 \times 7 = 56$   
 $8 \times 8 = 64$   
 $8 \times 9 = 72$

**59**  $9 \times 2 = 18$   
 $9 \times 3 = 27$   
 $9 \times 4 = 36$   
 $9 \times 5 = 45$   
 $9 \times 6 = 54$   
 $9 \times 7 = 63$   
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 $9 \times 9 = 81$

**60**  $2 \times 2 = 4$   
 $2 \times 3 = 6$   
 $2 \times 4 = 8$   
 $2 \times 5 = 10$   
 $2 \times 6 = 12$   
 $2 \times 7 = 14$   
 $2 \times 8 = 16$   
 $2 \times 9 = 18$

**61**  $3 \times 3 = 9$   
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 $3 \times 5 = 15$   
 $3 \times 6 = 18$   
 $3 \times 7 = 21$   
 $3 \times 8 = 24$   
 $3 \times 9 = 27$

**62**  $4 \times 4 = 16$   
 $4 \times 5 = 20$   
 $4 \times 6 = 24$   
 $4 \times 7 = 28$   
 $4 \times 8 = 32$   
 $4 \times 9 = 36$

**63**  $5 \times 5 = 25$   
 $5 \times 6 = 30$   
 $5 \times 7 = 35$   
 $5 \times 8 = 40$   
 $5 \times 9 = 45$

**64**  $6 \times 6 = 36$   
 $6 \times 7 = 42$   
 $6 \times 8 = 48$   
 $6 \times 9 = 54$

**65**  $7 \times 7 = 49$   
 $7 \times 8 = 56$   
 $7 \times 9 = 63$

**66**  $8 \times 8 = 64$   
 $8 \times 9 = 72$

**67**  $9 \times 9 = 81$

**68**  $2 \times 2 = 4$   
 $2 \times 3 = 6$   
 $2 \times 4 = 8$   
 $2 \times 5 = 10$   
 $2 \times 6 = 12$   
 $2 \times 7 = 14$   
 $2 \times 8 = 16$   
 $2 \times 9 = 18$

**69**  $3 \times 3 = 9$   
 $3 \times 4 = 12$   
 $3 \times 5 = 15$   
 $3 \times 6 = 18$   
 $3 \times 7 = 21$   
 $3 \times 8 = 24$   
 $3 \times 9 = 27$

**70**  $4 \times 4 = 16$   
 $4 \times 5 = 20$   
 $4 \times 6 = 24$   
 $4 \times 7 = 28$   
 $4 \times 8 = 32$   
 $4 \times 9 = 36$

**71**  $5 \times 5 = 25$   
 $5 \times 6 = 30$   
 $5 \times 7 = 35$   
 $5 \times 8 = 40$   
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**72**  $6 \times 6 = 36$   
 $6 \times 7 = 42$   
 $6 \times 8 = 48$   
 $6 \times 9 = 54$

**73**  $7 \times 7 = 49$   
 $7 \times 8 = 56$   
 $7 \times 9 = 63$

**74**  $8 \times 8 = 64$   
 $8 \times 9 = 72$

**75**  $9 \times 9 = 81$

**76**  $2 \times 2 = 4$   
 $2 \times 3 = 6$   
 $2 \times 4 = 8$   
 $2 \times 5 = 10$   
 $2 \times 6 = 12$   
 $2 \times 7 = 14$   
 $2 \times 8 = 16$   
 $2 \times 9 = 18$

**77**  $3 \times 3 = 9$   
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 $3 \times 5 = 15$   
 $3 \times 6 = 18$   
 $3 \times 7 = 21$   
 $3 \times 8 = 24$   
 $3 \times 9 = 27$

**78**  $4 \times 4 = 16$   
 $4 \times 5 = 20$   
 $4 \times 6 = 24$   
 $4 \times 7 = 28$   
 $4 \times 8 = 32$   
 $4 \times 9 = 36$

**79**  $5 \times 5 = 25$   
 $5 \times 6 = 30$   
 $5 \times 7 = 35$   
 $5 \times 8 = 40$   
 $5 \times 9 = 45$

**80**  $6 \times 6 = 36$   
 $6 \times 7 = 42$   
 $6 \times 8 = 48$   
 $6 \times 9 = 54$

**81**  $7 \times 7 = 49$   
 $7 \times 8 = 56$   
 $7 \times 9 = 63$

**82**  $8 \times 8 = 64$   
 $8 \times 9 = 72$

**83**  $9 \times 9 = 81$

**84**  $2 \times 2 = 4$   
 $2 \times 3 = 6$   
 $2 \times 4 = 8$   
 $2 \times 5 = 10$   
 $2 \times 6 = 12$   
 $2 \times 7 = 14$   
 $2 \times 8 = 16$   
 $2 \times 9 = 18$

**85**  $3 \times 3 = 9$   
 $3 \times 4 = 12$   
 $3 \times 5 = 15$   
 $3 \times 6 = 18$   
 $3 \times 7 = 21$   
 $3 \times 8 = 24$   
 $3 \times 9 = 27$

**86**  $4 \times 4 = 16$   
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 $4 \times 6 = 24$   
 $4 \times 7 = 28$   
 $4 \times 8 = 32$   
 $4 \times 9 = 36$

**87**  $5 \times 5 = 25$   
 $5 \times 6 = 30$   
 $5 \times 7 = 35$   
 $5 \times 8 = 40$   
 $5 \times 9 = 45$

**88**  $6 \times 6 = 36$   
 $6 \times 7 = 42$   
 $6 \times 8 = 48$   
 $6 \times 9 = 54$

**89**  $7 \times 7 = 49$   
 $7 \times 8 = 56$   
 $7 \times 9 = 63$

**90**  $8 \times 8 = 64$   
 $8 \times 9 = 72$

**91**  $9 \times 9 = 81$

## Multiplication Tables

x	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

**26**  $2 \times 2 = 4$   
 $2 \times 3 = 6$   
 $2 \times 4 = 8$   
 $2 \times 5 = 10$   
 $2 \times 6 = 12$   
 $2 \times 7 = 14$   
 $2 \times 8 = 16$   
 $2 \times 9 = 18$

**27**  $3 \times 3 = 9$   
 $3 \times 4 = 12$   
 $3 \times 5 = 15$   
 $3 \times 6 = 18$   
 $3 \times 7 = 21$   
 $3 \times 8 = 24$   
 $3 \times 9 = 27$

**28**  $4 \times 4 = 16$   
 $4 \times 5 = 20$   
 $4 \times 6 = 24$   
 $4 \times 7 = 28$   
 $4 \times 8 = 32$   
 $4 \times 9 = 36$

**29**  $5 \times 5 = 25$   
 $5 \times 6 = 30$   
 $5 \times 7 = 35$   
 $5 \times 8 = 40$   
 $5 \times 9 = 45$

**30**  $6 \times 6 = 36$   
 $6 \times 7 = 42$   
 $6 \times 8 = 48$   
 $6 \times 9 = 54$

**24**  $2 \times 2 = 4$   
 $2 \times 3 = 6$   
 $2 \times 4 = 8$   
 $2 \times 5 = 10$   
 $2 \times 6 = 12$   
 $2 \times 7 = 14$   
 $2 \times 8 = 16$   
 $2 \times 9 = 18$

**25**  $3 \times 3 = 9$   
 $3 \times 4 = 12$   
 $3 \times 5 = 15$   
 $3 \times 6 = 18$   
 $3 \times 7 = 21$   
 $3 \times 8 = 24$   
 $3 \times 9 = 27$

**26**  $4 \times 4 = 16$   
 $4 \times 5 = 20$   
 $4 \times 6 = 24$   
 $4 \times 7 = 28$   
 $4 \times 8 = 32$   
 $4 \times 9 = 36$

**27**  $5 \times 5 = 25$   
 $5 \times 6 = 30$   
 $5 \times 7 = 35$   
 $5 \times 8 = 40$   
 $5 \times 9 = 45$

**28**  $6 \times 6 = 36$   
 $6 \times 7 = 42$   
 $6 \times 8 = 48$   
 $6 \times 9 = 54$

**22**  $2 \times 2 = 4$   
 $2 \times 3 = 6$   
 $2 \times 4 = 8$   
 $2 \times 5 = 10$   
 $2 \times 6 = 12$   
 $2 \times 7 = 14$   
 $2 \times 8 = 16$   
 $2 \times 9 = 18$

**23**  $3 \times 3 = 9$   
 $3 \times 4 = 12$   
 $3 \times 5 = 15$   
 $3 \times 6 = 18$   
 $3 \times 7 = 21$   
 $3 \times 8 = 24$   
 $3 \times 9 = 27$

**24**  $4 \times 4 = 16$   
 $4 \times 5 = 20$   
 $4 \times 6 = 24$   
 $4 \times 7 = 28$   
 $4 \times 8 = 32$   
 $4 \times 9 = 36$

**25**  $5 \times 5 = 25$   
 $5 \times 6 = 30$   
 $5 \times 7 = 35$   
 $5 \times 8 = 40$   
 $5 \times 9 = 45$

**26**  $6 \times 6 = 36$   
 $6 \times 7 = 42$   
 $6 \times 8 = 48$   
 $6 \times 9 = 54$

**82**  $2 \times 2 = 4$   
**83**  $3 \times 3 = 9$   
**84**  $4 \times 4 = 16$   
**85**  $5 \times 5 = 25$   
**86**  $6 \times 6 = 36$   
**87**  $7 \times 7 = 49$   
**88**  $8 \times 8 = 64$   
**89**  $9 \times 9 = 81$   
**90**  $10 \times 10 = 100$   
**91**  $11 \times 11 = 121$   
**92**  $12 \times 12 = 144$   
**93**  $13 \times 13 = 169$   
**94**  $14 \times 14 = 196$   
**95**  $15 \times 15 = 225$   
**96**  $16 \times 16 = 256$   
**97**  $17 \times 17 = 289$   
**98**  $18 \times 18 = 324$   
**99**  $19 \times 19 = 361$   
**100**  $20 \times 20 = 400$

**58**  $2 \times 116 = 232$   
**59**  $3 \times 174 = 522$   
**60**  $4 \times 232 = 928$   
**61**  $5 \times 290 = 1450$   
**62**  $6 \times 348 = 2088$   
**63**  $7 \times 406 = 2842$   
**64**  $8 \times 464 = 3712$   
**65**  $9 \times 522 = 4698$   
**66**  $10 \times 580 = 5800$   
**67**  $11 \times 638 = 7018$   
**68**  $12 \times 696 = 8352$   
**69**  $13 \times 754 = 9802$   
**70**  $14 \times 812 = 11368$   
**71**  $15 \times 870 = 13050$   
**72**  $16 \times 928 = 14848$   
**73**  $17 \times 986 = 16762$   
**74**  $18 \times 1044 = 18792$   
**75**  $19 \times 1102 = 20938$   
**76**  $20 \times 1160 = 23200$

**40**  $2 \times 80 = 160$   
**41**  $3 \times 120 = 360$   
**42**  $4 \times 160 = 640$   
**43**  $5 \times 200 = 1000$   
**44**  $6 \times 240 = 1440$   
**45**  $7 \times 280 = 1960$   
**46**  $8 \times 320 = 2560$   
**47**  $9 \times 360 = 3240$   
**48**  $10 \times 400 = 4000$   
**49**  $11 \times 440 = 4840$   
**50**  $12 \times 480 = 5760$   
**51**  $13 \times 520 = 6760$   
**52**  $14 \times 560 = 7840$   
**53**  $15 \times 600 = 9000$   
**54**  $16 \times 640 = 10240$   
**55**  $17 \times 680 = 11560$   
**56**  $18 \times 720 = 12960$   
**57**  $19 \times 760 = 14440$   
**58**  $20 \times 800 = 16000$

**62**  $2 \times 124 = 248$   
**63**  $3 \times 186 = 558$   
**64**  $4 \times 248 = 992$   
**65**  $5 \times 310 = 1550$   
**66**  $6 \times 372 = 2232$   
**67**  $7 \times 434 = 3038$   
**68**  $8 \times 496 = 3968$   
**69**  $9 \times 558 = 5022$   
**70**  $10 \times 620 = 6200$   
**71**  $11 \times 682 = 7502$   
**72**  $12 \times 744 = 8928$   
**73**  $13 \times 806 = 10480$   
**74**  $14 \times 868 = 12152$   
**75**  $15 \times 930 = 13950$   
**76**  $16 \times 992 = 15872$   
**77**  $17 \times 1054 = 17918$   
**78**  $18 \times 1116 = 19888$   
**79**  $19 \times 1178 = 22382$   
**80**  $20 \times 1240 = 24800$

**36**  $2 \times 72 = 144$   
**37**  $3 \times 108 = 324$   
**38**  $4 \times 144 = 576$   
**39**  $5 \times 180 = 900$   
**40**  $6 \times 216 = 1296$   
**41**  $7 \times 252 = 1764$   
**42**  $8 \times 288 = 2304$   
**43**  $9 \times 324 = 2916$   
**44**  $10 \times 360 = 3600$   
**45**  $11 \times 396 = 4356$   
**46**  $12 \times 432 = 5184$   
**47**  $13 \times 468 = 6084$   
**48**  $14 \times 504 = 7056$   
**49**  $15 \times 540 = 8100$   
**50**  $16 \times 576 = 9216$   
**51**  $17 \times 612 = 10404$   
**52**  $18 \times 648 = 11664$   
**53**  $19 \times 684 = 13002$   
**54**  $20 \times 720 = 14400$

**62**  $2 \times 124 = 248$   
**63**  $3 \times 186 = 558$   
**64**  $4 \times 248 = 992$   
**65**  $5 \times 310 = 1550$   
**66**  $6 \times 372 = 2232$   
**67**  $7 \times 434 = 3038$   
**68**  $8 \times 496 = 3968$   
**69**  $9 \times 558 = 5022$   
**70**  $10 \times 620 = 6200$   
**71**  $11 \times 682 = 7502$   
**72**  $12 \times 744 = 8928$   
**73**  $13 \times 806 = 10480$   
**74**  $14 \times 868 = 12152$   
**75**  $15 \times 930 = 13950$   
**76**  $16 \times 992 = 15872$   
**77**  $17 \times 1054 = 17918$   
**78**  $18 \times 1116 = 19888$   
**79**  $19 \times 1178 = 22382$   
**80**  $20 \times 1240 = 24800$

**82**  $2 \times 164 = 328$   
**83**  $3 \times 246 = 738$   
**84**  $4 \times 328 = 1012$   
**85**  $5 \times 410 = 2050$   
**86**  $6 \times 492 = 2952$   
**87**  $7 \times 574 = 4018$   
**88**  $8 \times 656 = 5248$   
**89**  $9 \times 738 = 6642$   
**90**  $10 \times 820 = 8200$   
**91**  $11 \times 902 = 9922$   
**92**  $12 \times 984 = 11808$   
**93**  $13 \times 1066 = 13858$   
**94**  $14 \times 1148 = 16072$   
**95**  $15 \times 1230 = 18450$   
**96**  $16 \times 1312 = 21000$   
**97**  $17 \times 1394 = 23700$   
**98**  $18 \times 1476 = 26568$   
**99**  $19 \times 1558 = 29602$   
**100**  $20 \times 1640 = 32800$

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Cut out and paste over the existing front cover, before making multiple copies.

**A Book of  
Multiplication  
Tables  
10 to 99**

