Calendar Models

I made my first calendar model, based on the regular dodecahedron, after reading about it in Martin Gardner's famous column in the *Scientific American*. (Late 70's?) I then went on to 'calendarise' other models. I made them available for others to photocopy and use, in the November 1984 editon of Mathematics in School, and it became an annual event thereafter. The notes which follow are based upon my own experiences over the years, plus comments received from other teachers who used the models in their classrooms. The models are undoubtedly popular.

As with all model-making the accuracy of the net which is used to make the model is of paramount importance, and that must be kept in mind when reproducing the material offered here.

On all of the sheets you will find a small cross drawn in each corner. These mark out an oblong which was originally 190 millimetres by 277 millimetres. Whether it is exactly that in your case (on the master copy or photo copy) you will have to find out. If not (a +/-1 mm variation hardly matters) check the ratio. If that is 1.458 or close, then the net will do. Of course, with some models it matters more than with others. One tip. If you are using Acrobat Reader version 4.0 (and you should be by now) when you print, select the 'Print as Image' option. You do not have that choice in version 3.0. The real test of course is to take the model through to completion and see that it actually does fit together properly. But it all takes a lot of time.

For best results use a laser printer at 600 dpi (or better) and a good paper. However, even with an ink-jet at 300 dpi the results are very satisfactory.

The card should be 160 gsm (grams/square metre) which seems to be about the limit for ordinary photocopiers and is quite robust enough for the models. In case you did not know, the paper normally used in photocopiers is about 80 gsm. Thicker card can give problems since the nets have no allowance in them for losses in folding. For the glueless models, a 100 gsm paper is good (and cheaper).

Do make sure all scoring is done **before** doing any glueing. It is extremely difficult to do it in the middle of making the model. And an edge which has not been scored and creased can spoil the crisp clean look of the complete model.

Think about personalising it before printing off several hundred. This might take the form of marking off important dates (by drawing a ring around them) like terms and holidays, or putting in the name and phone number of the school. Or, for a very limited edition, the names of all the pupils in that class. Of course just what can be done will depend upon there being space, and your ingenuity (to say nothing of your facilities, time, interest and patience)!

General Notes on Calendars and Models

It is best if they can be reproduced on card of course, but satisfactory results can be obtained with paper, though the finished product is naturally more fragile. Whatever the material, it is best to have a variety of colours, but not too dark for reasons of legibility.

These models are so attractive that it is possible to expect them to purchased, you don't have to give them away. The cost of printing them will vary enormously, depending upon the process used as well as the price. Whether you give them away, sell them, and what price you sell them at, will probably be determined by the relative affluence of your environment (department and pupils).

It might help to know how other schools organise this activity. One makes an estimate of requirements (very difficult this the first time around) plus some extra for spares, and has the lot printed up by offset-litho. Decisions are made, by individual teachers, as to which lesson period will be used for calendar-making and prior notice is given to the class. This takes the form of showing some already made-up examples, saying what charge (if any) is to be made, and that scissors and glue will be required. As an added incentive you might tell them that all those not engaged in the activity will have to do pages of sums!

Some schools merely sell the sheets to their pupils for taking away and making-up in their own time. Others use it as a Maths Club activity. Once started, it can soon become an annual event that everyone looks forward to.

Since this model is likely to be actually used and handled, it might be worthwhile to think about giving it a little added weight. This can most easily be done before sealing down the last face by putting in a quantity of dried peas - about half to three-quarters full. This is also a help in ensuring that the last face is properly stuck down - put the glue on the flaps, press the face into position and then carefully invert the model upon a flat surface. I am still re-cycling the peas I started with many years ago, each year taking the peas out of the old model and putting them into the new one. These models then serve well as paper-weights (or maracas).

Several pupils have, at different times and in different schools, come up with the idea of using the model as a container for a small present, leaving the last face unglued and fastening it with a pretty ribbon. This is then given to someone, together with an offer to glue the last face in place once the contents have been removed. The most common filling seems to be small (wrapped) sweets. In fact, whether filled or not, many of these models have been given as presents - and you can't do that with much of what comes out of a mathematics classroom!

Of course some work could be done on the calendar itself. For instance, in the context of calendar-making, how many different calendars are there? Well, January 1st must start on one day out of seven, and the year must be either a leap-year or not. So we see that there are 14 (= 7 × 2) different calendars. {Incidentally, we are talking ‘Gregorian’ here and not ‘Julian’, ‘Coptic’, ‘Islamic’, ‘Hebrew’ or any other variety.} So, having determined how many there are, identify them with a code letter, and then write out a list of years, say from 1900 to 2050, and write against each one its relevant code letter. You are then halfway to creating a perpetual calendar. The list is also worth looking at for patterns and frequencies - some years are ‘used’ much more than others.

And then there are the ‘named’ days. These are of two sorts. One is the fixed variety like St. Valentine’s Day, St. George’s Day, St. David’s Day, Mid-summer’s Day, Christmas Day and so on. The others vary from year to year and are fixed in accordance with some well-defined rules - what are the rules? These are days like Good Friday, May Day, Spring Bank Holiday, Pancake Day (why that name?) and so on. Many are to do with religious observances and are fixed from the date of Easter, and that could lead into the mathematics involved in finding out the date of Easter in any given year. Or there is the problem of establishing the day of the week on which any particular date falls.

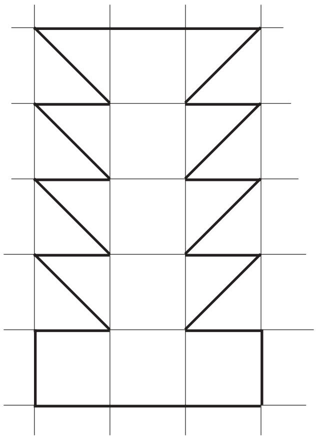
And, of course, there is the matter of the other types of calendar still in use with different ethnic groups. But be warned, though interesting to read about, most of them have structures which are quite difficult to follow and fiendish to ‘mathematicise’.

Whatever lines you choose to follow, the whole thing can soon take on the complexion of a major investigation, and the making of the calendar models will provide welcome relief!

The following pages deal with the actual models themselves.

Glueless modelling

For me, the work on models of this type started in 1973 with a series of articles by James Brunton[1] about paper-folding, which included many geometrical ideas and also an account of how to make models which required no sticking to be done. The simplest case is making a cube, and a demonstration of this makes a good starting point.



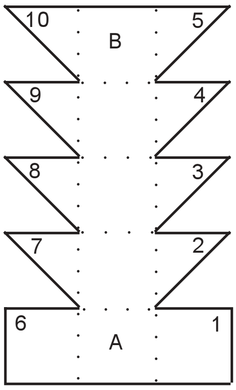
Using either paper or card, on a 5 by 3 grid of squares, mark out the shape shown on the left. 2.5 cm is a good working size for the squares. Cut out the shape shown in outline by the heavy lines and mark it as shown on the diagram below.

Score and crease along the dotted lines. Use the horizontal creased edges to bend the shape around so that face A goes under face B.

Fold down flaps 1, 2, 3 and 4 in that order. Tuck 5 under 2.

Fold down flaps 6, 7, 8 and 9 in that order. Tuck 10 under 7.

You should now have a cube that does not readily come apart.



A task that can be set, once the making of a cube by this method has been mastered, is to require dots to be put on so that the finished cube represents a die. The specification should insist that it models exactly a conventional die as to the placing of the dots.

Having seen the principles underlying this method of assembly it is not difficult to adapt it to other models. The next simplest is probably the square-based pyramid for which the base can be modelled on one-half of the cube. One feature of this type of modelling is that the net always has one extra face compared with the more usual net for that shape. This is the overlap required to cover the gap which would otherwise appear between two edges.

It is clearly not difficult to make a cuboid which has a square cross-section, but a cuboid having 3 different dimensions will need a little head-scratching to get the end flaps right. A considerable amount of 'trial and improvement' is needed and, to save cost, is best done on paper before moving on to a card model. The triangular prism calls for another solution to the design of the end flaps, but this solution can then be used in making a tetrahedron. After that it is a matter of interest and perseverance. The hexagonal prism (on which a calendar is printed) is based on these principles. You might like to add additional information to the end panels before copying it - like dates of school holidays or the school telephone number. This will need some careful placing if it is to be seen after assembly.

With a clear idea of what glueless packaging means it is interesting to seek out examples of its commercial uses. For reasons of security some glueing has to be allowed, but the principle can be observed. Perhaps the best known example is the hexagonal prism used to hold a well-known brand of cleaning cloths, and the exactness and ease with which the two ends work make it a joy to handle. Unfortunately the precision needed to employ the interlocking principle used in that is not easily obtainable in the classroom.

There are many other examples to be found on the shelves of any supermarket. Several of the goodies produced for Christmas and Easter are often presented in packages of this type. The contents of course are not necessary and will have to be disposed of - could research in the name of mathematics always be so pleasurable? Apart from the total package, it is worthwhile looking at the bottoms of some more conventional boxes which are also often glueless. It allows the box to be folded flat for storage purposes, but to be quickly opened up to make a rigid container with a firm bottom through which the contents will not slip. The ingenuity and variety found in making this possible is very interesting.

All of this work will require that a considerable amount of practical geometry is covered and ensure that much spatial thinking is done. It can be enjoyable too!

Notes

1. Mathematical Exercises in paper folding, *Mathematics in School,* Vol.2 No.4 (July 1973) and continued into the 7 subsequent editions of the journal. It is a great pity that the series never appears to have been put into a more permanent form.

Plaited models

Plaiting *(pronounced "platting")* is another way of making models without using any glue, and it is used for making just one of the calendar models - the Tetrahedron, which is the easiest model of all to make with this technique.

For these models, paper is definitely better than card. In fact it is essential if making anything more complex than the tetrahedron.

Instructions for making a cube are given below. Whether the net given is to be copied (as it says) or itself cut up and used is a matter of choice. Making a copy has some merit in the learning situation, and a master copy of suitable 20 millimetre squared paper is provided with these notes. There is also a sheet of 30 mm squared paper for those with less nimble fingers.

As an additional task, make a plaited cube to serve as a die. Easy, put dots on in place of the numbers. But, given the requirement that the opposite faces of a die must add up to a total of 7, some adjustment will have to be made.

For those wishing to know more about plaiting the standard reference on the subject is

Plaited Polyhedra by A R Pargeter

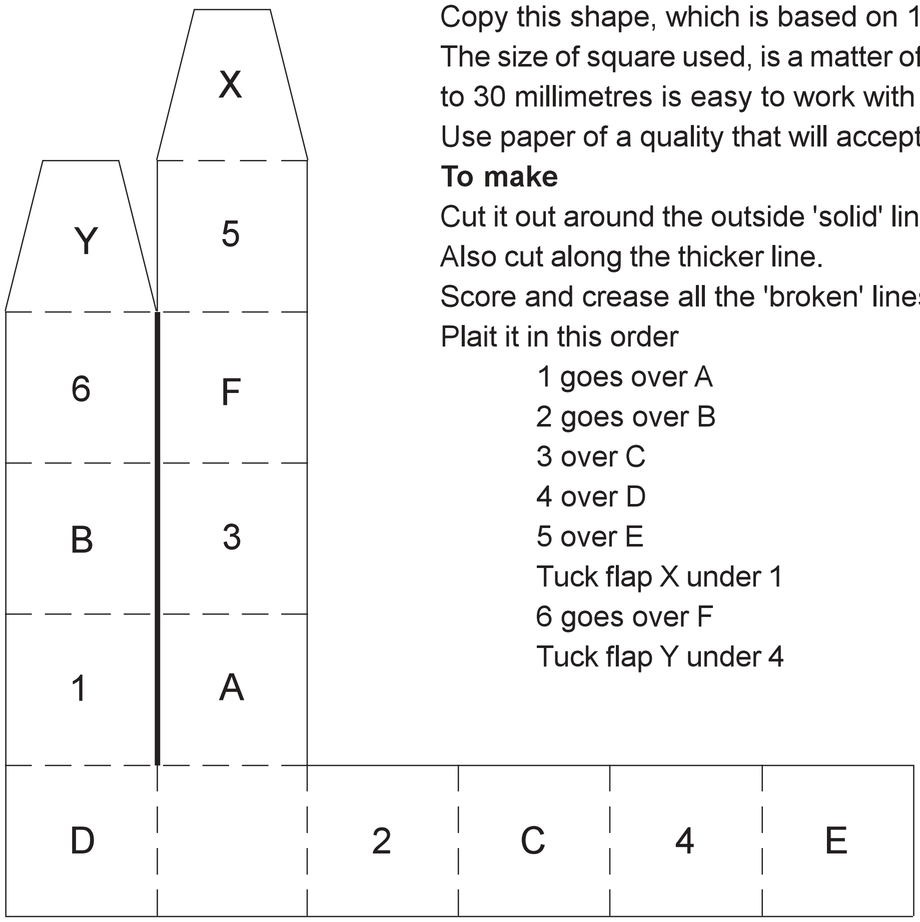
which was published in the

Mathematical Gazette May 1959 (Vol. 43 No. 344) pages 88 to 101 which gives the nets and instructions for many models from a tetrahedron to the great stellated dodecahedron.

Interestingly, in that article he refers to an earlier work of 1888, in which that author states he first made such models 40 years previously. The idea has been around for some time!

It certainly is a fascinating one.

Plaiting a Cube



Copy this shape, which is based on 15 squares, on to paper.
The size of square used, is a matter of choice but, anything from 20 to 30 millimetres is easy to work with in the assembly stage.
Use paper of a quality that will accept scoring.

To make

Cut it out around the outside 'solid' line.
Also cut along the thicker line.
Score and crease all the 'broken' lines.
Plait it in this order

1 goes over A
2 goes over B
3 over C
4 over D
5 over E
Tuck flap X under 1
6 goes over F
Tuck flap Y under 4

Deltahedrons

A deltahedron[1] is a polyhedron whose faces are all equilateral triangles. The simplest possible case is the regular tetrahedron. Then, using several of these as 'building blocks' it is clearly possible to keep putting them together, face to face, to make an infinity of different deltahedrons. Whilst such developments have their own fascination we will concern ourselves here only with convex models. (A convex polyhedron is one in which ALL the diagonals, joining one vertex to another, must lie on or within the polyhedron. Alternatively, it is one in which NO internal dihedral angle between adjacent faces is greater than 180º.)

It is easy to visualise the case where putting two tetrahedrons together makes a deltahedron having 6 faces which is a hexadeltahedron (or a deltahexahedron). The next convex deltahedron, having 8 faces, is the octadeltahedron which is better known as the regular octahedron. Note that the *regular* octahedron cannot be built from tetrahedrons, though a concave *non-regular* octahedron can.

We have 4, 6 and 8 faces as being possible so far. What other possibilities might there be? We then remember the regular icosahedron so know it can be done for 20 faces.

As a digression, since we now have 4, 6, 8 and 20 in our collection, we might wonder if only even numbers are possible and, if wishing to dwell upon that, want to be aware of an argument about it. Each face has 3 edges, so any deltahedron with f faces should have $3f$ edges. But each edge of a polyhedron is actually made by joining 2 face edges and so a deltahedron of f faces will have $3f/2$ edges. If f is odd then we need to produce a polyhedron having half an edge and, in order to satisfy Euler's law (faces + vertexes - edges = 2), also half a vertex. Until such a model is produced we will assume that a deltahedron must have an even number of faces.

Now, 4 is obviously the smallest and, to save wasting time, we will admit that 20 is known to be the biggest possible. Are all the other numbers (10, 12, 14, 16, 18) possible? Since there are only five cases it is not a major task to attempt to make them and try to discover which ones cannot be done. Alternatively, it could be tackled by stating that it has been proved[2] (but not until 1947) that only eight convex deltahedrons exist. We have identified four of them so only need to find the other four from our five possibilities listed above.

Fortunately any net of a deltahedron must be contained on an isometric grid, which simplifies the task enormously. The grid-size used on normal isometric paper is a little small however, and makes the cutting and folding fiddly. Counting small triangles together to make bigger ones proves to be rather confusing for many and so, to help with this investigation, a master grid of a suitable size has been provided further on in these notes for photocopying. As a starting point it is a good idea to require nets to be found for the 4, 6 and 8 cases. That is not difficult.

As another digression it could be asked how many different nets there are for the regular octahedron. There are 11 and, recalling that there are also 11 for the cube, plus the fact that the cube and octahedron are duals, we pause to wonder if those two facts are linked or merely coincidental?

Let us now admit that one of the possible cases has 12 faces, which means that it is suited to being the basis of a calendar. That is the delta dodecahedron used for the calendar model provided. Its one drawback is the fact that on four edges the internal dihedral angle between the two adjacent faces is not far removed from 180º and so it is easy to distort the model. The big surprise to me was the stability of the model, which means it can always be stood so that the current month is in a good viewing position the right way up at the top. It makes a suitable conclusion to a worthwhile journey. No, I have not forgotten anything, but leave for you the task of establishing the other three convex deltahedrons (from 10, 14, 16 or 18 faces) that can be made. Or, which one cannot be made, if you prefer.

Notes

1. The name was first proposed and used in *Mathematical Models* by Cundy & Rollett, O.U.P. 1951

2. A good account is contained in *Excursions in Mathematics,* Beck, Bleicher & Crowe, Worth Publishers, N.Y. 1969

Other Models

Prisms

With three different prisms (the triangular, rhombic, and hexagonal) on display the underlying structure of a prism (parallel similar end-faces which give it a uniform cross-section) can be pointed out. Also the the way the name of that end-face determines the name of the prism. If the faces joining the two end-faces are rectangles then it is a ‘right prism’ - because those faces must be at right-angles to the end-faces. Do not forget that a cuboid (having either square or oblong-shaped end faces) is also a prism and moreover, can be considered as such in three different directions. One practical point with making these. That long tab can be troublesome to glue. Put a ruler down through the model and press against that.

Antiprisms

Very similar to prisms but with one important difference. The two parallel polygonal end-faces are still needed but now they are rotated relative to each other. This then means that in order to connect their edges with plane (flat) shapes, as is required for a polyhedron, triangles must be used instead of rectangles or parallelograms. Difficult to describe and imagine but, once seen never forgotten. One of these should always be seen by anyone who is interested in prisms.

Tetrahedron

This a shape which disappoints me - I always feel it ought to tessellate in 3-space (just as the triangle does in 2-space) - but the regular one doesn't. You could try to demonstrate this, but it requires considerable dexterity - having three hands helps. It is possible to make an irregular one that will tessellate.

It might be worth pointing that a tetrahedron is also a pyramid, a triangular-based pyramid. Or, more specifically, since this is a regular tetrahedron, it is a right, triangular-based pyramid.

Pyramid

Say 'pyramid' and it means only one thing to most people - those shapes the Pharoahs used to bury themselves in. Since we never (what never?) see or use any other type it is not surprising. However, mathematically we need to identify the base shape and whether it is 'right' or 'oblique', so the opportunity could be taken to make that point here.

Oblique Pyramid

This is a pyramid whose apex is not located on the vertical through the centre of the base. The model offered has the apex over one corner of the base. The perpendicular height is such that the whole thing is exactly one-third of a cube. This particular shape is also known as a 'yangma'.

Get three. Put them together to make a cube. Demonstrate the truth of the formula for finding its volume.

Rhombohedron

This is a shape which is commonly seen in crystals, and is often called a rhomboid (which, unfortunately, is also the name of a particular 2-dimensional shape). It is a pity that it requires a ‘stand’ to display it properly, though this might be done as a separate project if the given net is not used.

Octahedron

This is more of a novelty than a model of any mathematical importance, but it has a certain charm. Perhaps it might be important to stress that it is not a regular octahedron - it would be useful to have one of those around if possible. Or just show what it looks like like by putting two square-based pyramids together.

Flexi-tetrahedron

Easy to make the necessary 'band' but many people will have a little difficulty in folding it up into a tetrahedron. Though once it has been done you wonder why there was any problem. This is a well-known puzzle model in recreational mathematics (origin not known).

However, it was Liz Meenan, who is an Education Officer for television's Channel 4, who actually fitted a calendar on to it. Thank you Liz.

Dodecahedrons

There are three dodecahedrons available.

The **regular**, in spite of the fact that it is the hardest and most time-consuming to make, is always the most popular (and satisfying). This one can be scored and folded on its centre-line to make a novelty card, which someone else can make up!

Given that three (or more) of the Rhombic Dodecahedrons are made up, it is easy to demonstate that it does tessellate in 3-space. (A fact which surprises most people.) It also easy to show that the other two dodecahedrons come nowhere near it.

Cone

The (truncated) cone is one of the simplest to make, with only one tab to be glued. However, it does provide one difficulty not present in other models in that it wants to 'spring apart'. This can be overcome by rolling it rather tightly first, before attempting to glue it. Printing it on paper rather than card also makes it a little easier, and it is still reasonably robust.

It is truncated because it is very difficult to roll and glue a cone with an apex. But having the hole at the top also make it easy to pick up and turn.

For a bit of frivolity, put a head on it! Get a table-tennis (or golf) ball, draw a face on it, and it will rest in the hole.

Calendar Cubes

This could well be started off as a Design Problem.

> "How can you display all the numbers from 1 to 31 on two cubes with only one digit allowed on each face?"

It is a good one, easy to state and to understand, but requiring one insightful moment during its solving. After that the sheets of printed nets can be shown and used.

Then they need to be displayed. Well that could be a complete 'design and make' project. There is a drawing on the sheet to convey the idea of what is wanted. Mind, even if the pre-printed sheet for the stand is used, it is not a job for the dextrously-challenged.

The attractiveness of the complete assembly can be enhanced by printing on a variety of colours, and then doing some swopping of the various elements between those who are making them. (The swopping should be done **before** the bits are cut and creased!)

Wall calendars

Clearly not mathematical models but these can be interesting and useful in another way.

The **Teddy Bears** calendar has its own set of notes.

The **Geometric** calendar offers scope for a variety of pattern work, requiring quite a bit of design and colouring to make it look at all attractive. The simple black and white examples offered on the last pages are intended for those who find it difficult to get started with their own ideas.

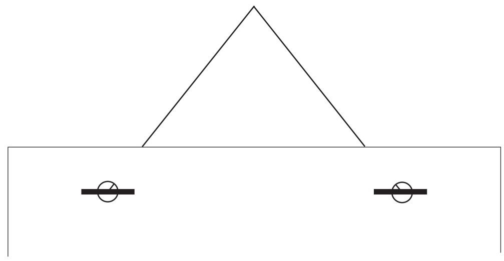
The **Anniversaries** calendar is similar to the previous one in that it presents more patterns but now there is some additional material to which attention could be drawn, and possibly used as the basis for some other work. Even something so simple as finding out who the people were (on a month by month basis) would be of benefit to many.

Hanging of Wall calendars

With these wall calendars made up of multiple sheets, they need to be collated and fastened together. One way is to use 3 or 4 staples along the top to bind the whole lot together.

A better method though is to punch 2 holes at the top of each of each sheet and then fasten them together with a 'treasury tag' of an appropriate size. [This is a piece of coloured string with a metal or plastic tag at each end which is secured at right-angles to the string.]

Thread the separate sheets on so that the tags are at the front of the calendar. The surplus string at the back then provides a ready-made hanger. Another advantage is that completed months do not have to be torn off but can be unthreaded and removed neatly.



The **Single Sheet** calendar requires no additional work at all. It is ready to go straight up on the wall or, even into an A4 ring-binder.

Pocket calendars

The pocket calendars are excellent for personalisation purposes. It means printing on both sides has to be organised but, printed (4 to a sheet) on card, cut out and folded, they can be used as a useful 'giveaway' to convey much information - similar to the sort of thing commercial businesses do.