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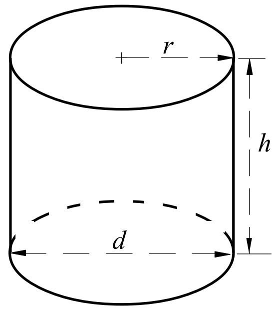
The full list of Contents is given opposite.
It is printed in black and red as shown here.
There are 44 pages + 4 preliminaries + cover.
Size is 145 mm by 210 mm. (about A5)

From the Web site mentioned above a Study Guide is available which gives familiarisation exercises, explanations on the use of formulas, worked examples, and further general exercises.

Cylinder



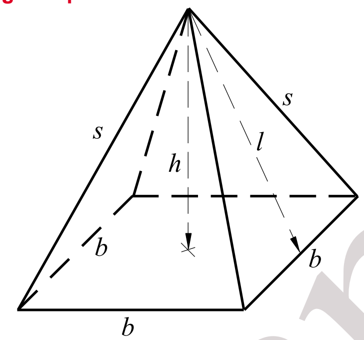
r = radius
h = height
V = volume
d = diameter
C = curved surface area *(without ends)*
T = total surface area *(with ends)*

$V = \pi \times r^2 \times h$ $\quad V = \pi \times d^2 \times h \div 4$ $\quad V = \dfrac{C \times r}{2}$

$C = 2 \times \pi \times r \times h$ $\quad C = \pi \times d \times h$ $\quad C = \dfrac{2 \times V}{r}$

$T = 2 \times \pi \times r \times (r + h)$

Pyramid
Right square-based



b = base edge
h = perpendicular height
V = volume
s = slant edge
l = slant height

$V = b^2 \times h \div 3$

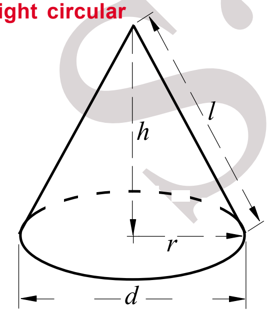
$h = 3 \times V \div b^2$ $\quad b = \sqrt{\dfrac{3 \times V}{h}}$

$h = \sqrt{s^2 - \dfrac{b^2}{2}}$ $\quad b = \sqrt{2 \times (s^2 - h^2)}$

$h = \sqrt{l^2 - \dfrac{b^2}{4}}$ $\quad b = 2 \times \sqrt{l^2 - h^2}$

$l = \sqrt{h^2 + \dfrac{b^2}{4}}$ $\quad s = \sqrt{h^2 + \dfrac{b^2}{2}}$

Cone
Right circular



r = radius of base
h = perpendicular height
l = slant height
d = diameter of base
C = curved area *(without base)*
V = volume

$V = \pi \times r^2 \times h \div 3$ $\quad V = \pi \times d^2 \times h \div 12$

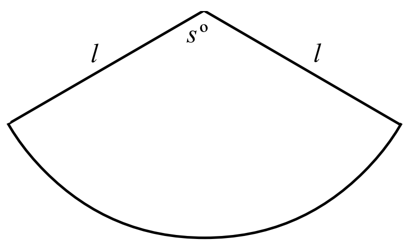
$C = \pi \times r \times l$ $\quad l = \sqrt{r^2 + h^2}$

$h = \sqrt{l^2 - r^2}$ $\quad r = \sqrt{l^2 - h^2}$

$h = \dfrac{3 \times V}{\pi \times r^2}$ $\quad r = \sqrt{\dfrac{3 \times V}{\pi \times h}}$

The correct size of sector, like that shown on the right, needed to make a particular cone, can be cut from a circle which has a radius of ***l*** and a sector angle of s° where

$$s^\circ = \frac{360 \times r}{l}$$



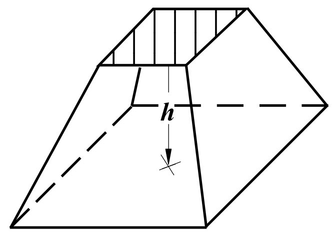
Frustum of a Pyramid

A, B = areas of the top and bottom parallel faces
h = perpendicular height between the two faces
V = volume

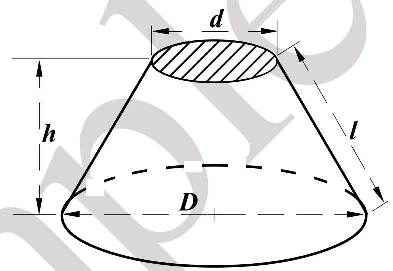
Note that the shapes of the top and bottom faces do not have to be squares, but they must be the same shape and parallel to each other.

$$V = (A + B + \sqrt{A \times B}\,) \times h \div 3$$

This formula also applies to the frustum of a cone.



Frustum of a Cone



D = diameter of base circle
d = diameter of top circle
h = perpendicular height between faces
l = slant height
C = curved surface area
V = volume

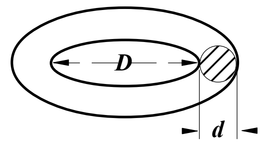
$V = \pi h (D^2 + Dd + d^2) \div 12$

$C = \pi l (D + d) \div 2$

$l = \left(\sqrt{(D - d)^2 + 4h^2}\right) \div 2$

Torus

A torus is a solid circular ring, made of material which has a circular cross-section. It is also known as an **anchor ring**.



D = diameter of inside of torus
d = diameter of circular cross-section
S = surface area
V = volume

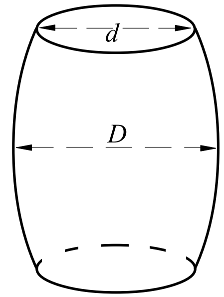
$V = \pi^2 d^2 (D + d) \div 4$

$S = \pi^2 d (D + d)$

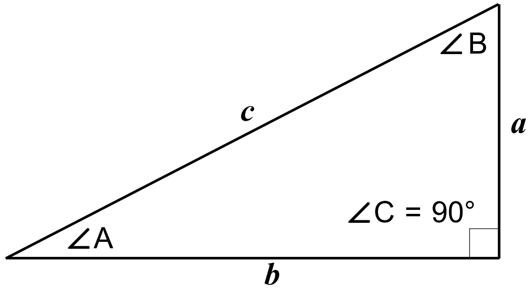
Barrel

A barrel, of the traditional type and made of wood, is roughly cylindrical in shape having two circular ends of the same diameter (d), bulging outwards to a bigger diameter (D) in the middle of its height (h), or length. It is difficult to calculate its exact volume (V), or capacity, but a very good approximation is

$$V = \pi h (39D^2 + 26Dd + 25d^2) \div 360$$



Right-angled Triangle



∠A = angle A, etc.
Sizes, in degrees, are given by *A, B, C.*

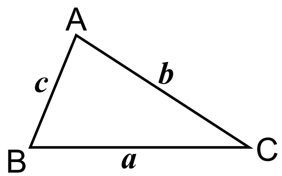
c is the hypotenuse.

Take care to match given data to the correct letters.

Given ↓	use the formula from the appropriate box below to find *a*	*b*	*c*	∠A	∠B
a b			$c = \sqrt{a^2 + b^2}$	$\tan A = \frac{a}{b}$	$\tan B = \frac{b}{a}$
a c		$b = \sqrt{c^2 - a^2}$		$\sin A = \frac{a}{c}$	$\cos B = \frac{a}{c}$
b c	$a = \sqrt{c^2 - b^2}$			$\cos A = \frac{b}{c}$	$\sin B = \frac{b}{c}$
a ∠A		$b = \frac{a}{\tan A}$	$c = \frac{a}{\sin A}$		$B = 90 - A$
a ∠B		$b = a \times \tan B$	$c = \frac{a}{\cos B}$	$A = 90 - B$	
b ∠A	$a = b \times \tan A$		$c = \frac{b}{\cos A}$		$B = 90 - A$
b ∠B	$a = \frac{b}{\tan B}$		$c = \frac{b}{\sin B}$	$A = 90 - B$	
c ∠A	$a = c \times \sin A$	$b = c \times \cos A$			$B = 90 - A$
c ∠B	$a = c \times \cos B$	$b = c \times \sin B$		$A = 90 - B$	

General Triangle

The semi-perimeter is
$s = (a + b + c) \div 2$
usually written as
$s = \frac{a+b+c}{2}$



The vertices of a triangle are identified as A, B, C, and the sizes of the angles of each of those vertices as *A, B, C*. The lengths of each of the edges opposite to those vertices are shown as *a, b, c*.

Δ is the symbol for **area**.

Area $= \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B = \frac{1}{2}bc\sin A = \sqrt{s(s-a)(s-b)(s-c)}$

Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule

$a^2 = b^2 + c^2 - 2bc\cos A$ *or* $\cos A = (b^2 + c^2 - a^2) \div 2bc$

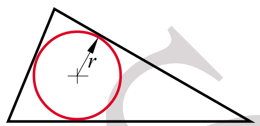
$b^2 = a^2 + c^2 - 2ac\cos B$ *or* $\cos B = (a^2 + c^2 - b^2) \div 2ac$

$c^2 = a^2 + b^2 - 2ab\cos C$ *or* $\cos C = (a^2 + b^2 - c^2) \div 2ab$

Half-angle Formulas

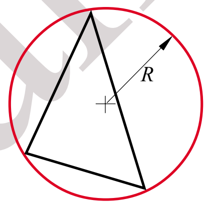
$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$ $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$ $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

Inscribed Circle



Radius $r = \frac{\Delta}{s}$

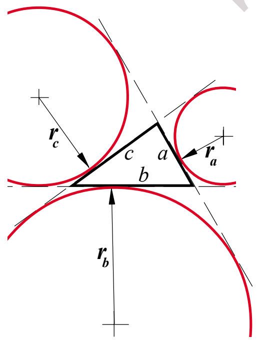
Circumscribed Circle



Radius $R = \frac{abc}{4\Delta}$

$R = \frac{a}{2\sin A}$ *or* $\frac{b}{2\sin B}$ *or* $\frac{c}{2\sin C}$

Escribed Circles

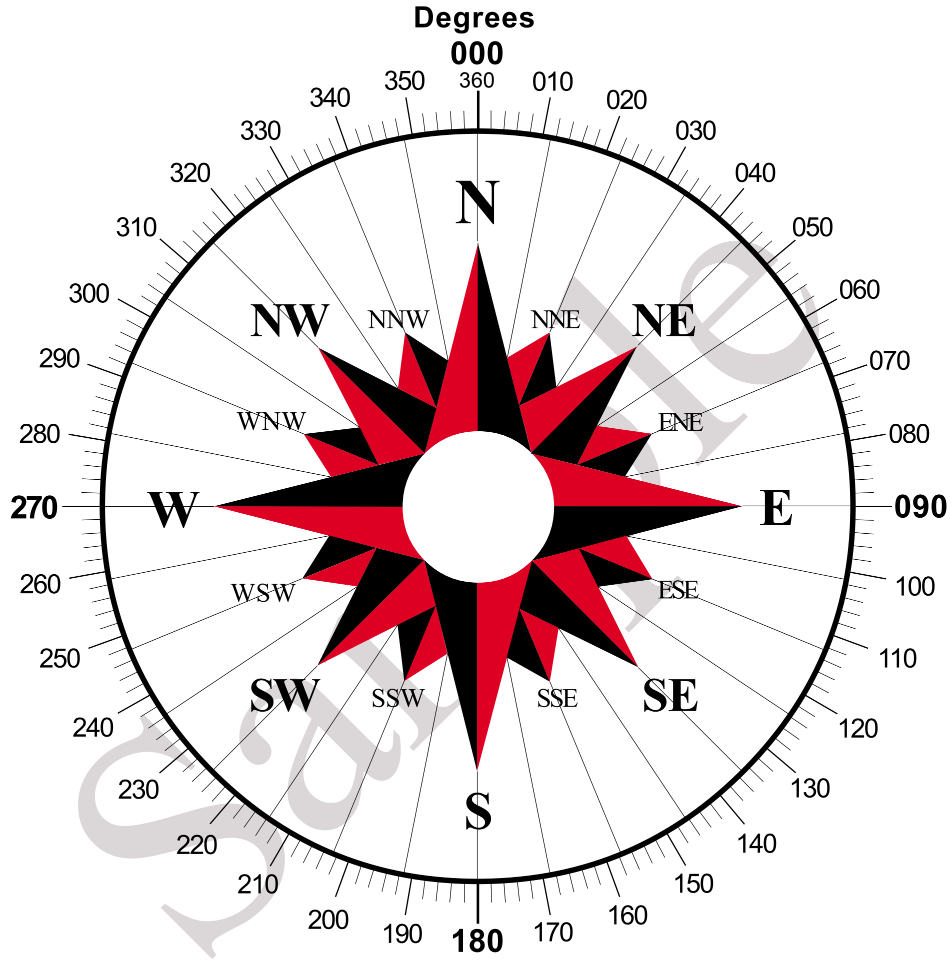


The different radii needed for the three possible escribed circles are identified by the letters of the edge on which each circle is placed.

$r_a = \frac{\Delta}{s-a}$ $r_b = \frac{\Delta}{s-b}$ $r_c = \frac{\Delta}{s-c}$

*All the formulas on this page are **cyclic**. That is, the six variables (a, b, c, A, B, C) can be changed around as long as the pattern of the formula is kept. This is seen in the **Cosine Rule** where all three possible variations are given, and the pattern is clear.*

Compass directions and 3-figure bearings

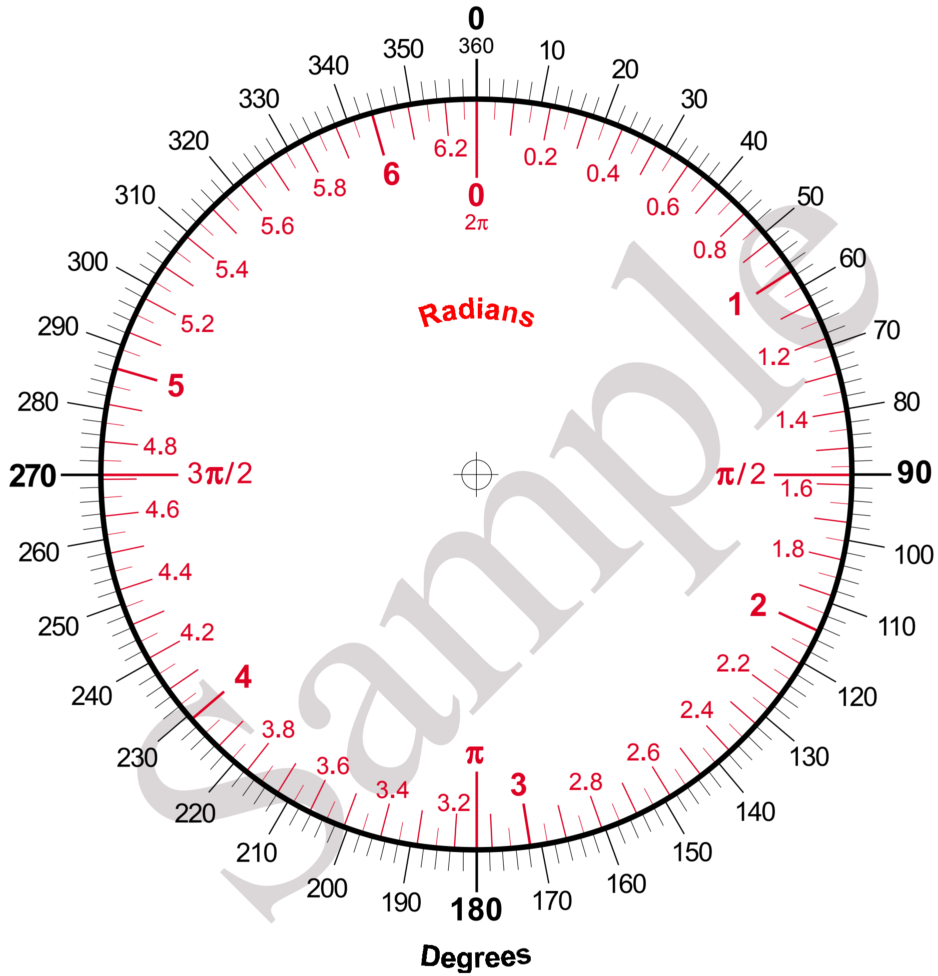


The four familiar compass directions (**N**orth, **E**ast, **S**outh, **W**est) originated in the 1300's though their names were different then. The directions between the four main ones were also named, creating the 16 points (North-east, South-southwest, etc.) as shown and named in the above diagram.

This meant directions could be given in 22.5° intervals which is rather large for navigation. Sub-divisions were made using directions like "Northeast by east" indicating a direction midway between NE and ENE, producing a total of 32 directions. These are shown as lines, but not named, in the diagram. The intervals between these were further divided into quarter-points (not shown above) so that direction intervals of a little under 3° were possible.

Note all directions (or bearings) in degrees should have 3 digits.

A conversion scale for radians and degrees



1 radian	=	180 ÷ π degrees	
1 rad	≈	57.295 779 513°	
1°	=	π/180	rad (≈ 0.017 453 293)
90°	=	π/2	rad (≈ 1.570 796 327)
180°	=	π	rad (≈ 3.141 592 654)
270°	=	3π/2	rad (≈ 4.712 388 980)
360°	=	2π	rad (≈ 6.283 185 307)

Trigonometry

$\operatorname{cosec} A = \dfrac{1}{\sin A}$ $\cot A = \dfrac{1}{\tan A}$

$\sec A = \dfrac{1}{\cos A}$ $\tan A = \dfrac{\sin A}{\cos A}$

Multiple angles

$\sin 2A = 2\sin A\cos A = \dfrac{2\tan A}{1+\tan^2 A}$

$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A$

$= 2\cos^2 A - 1 = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

$\sin 3A = 3\sin A - 4\sin^3 A$

$\cos 3A = 4\cos^3 A - 3\cos A$

$\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

Addition & product formulas

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

$\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$

$\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$

$\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$

$\cos B - \cos A = 2\sin\dfrac{A+B}{2}\sin\dfrac{B-A}{2}$

$\sin(A+B) + \sin(A-B) = 2\sin A\cos B$

$\sin(A+B) - \sin(A-B) = 2\cos A\sin B$

$\cos(A+B) + \cos(A-B) = 2\cos A\cos B$

$\cos(A-B) - \cos(A+B) = 2\sin A\sin B$

Identities

$\sin^2 A + \cos^2 A = 1$

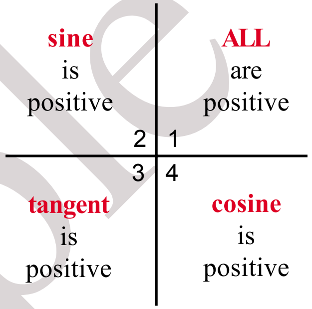
$\sec^2 A = 1 + \tan^2 A$

$\operatorname{cosec}^2 A = 1 + \cot^2 A$

1 radian $= 180 \div \pi$ **degrees**

1 rad $\approx 57.295\,779\,513°$

Angles in any quadrant



Negative angles

$\sin(^-A) = {}^-\sin A$

$\cos(^-A) = \cos A$

$\tan(^-A) = {}^-\tan A$

Half-angle formulas

$\cos A = \cos^2(\tfrac{A}{2}) - \sin^2(\tfrac{A}{2})$

$\sin A = 2\sin(\tfrac{A}{2})\cos(\tfrac{A}{2})$

$\tan A = \dfrac{2\tan(\frac{A}{2})}{1-\tan^2(\frac{A}{2})}$

$1 + \cos A = 2\cos^2(\tfrac{A}{2})$

$1 - \cos A = 2\sin^2(\tfrac{A}{2})$

If $t = \tan(\tfrac{A}{2})$ then

$\sin A = \dfrac{2t}{1+t^2}$

$\cos A = \dfrac{2t}{1-t^2}$

$\tan A = \dfrac{1-t^2}{1+t^2}$

Algebra

Sums of Powers of Natural Numbers

The first n natural numbers are

1, 2, 3, 4, 5, 6, 7, …… n

Their sum when each has been raised to the power k is

$\Sigma i^k = 1^k + 2^k + 3^k + 4^k + 5^k + 6^k + \ldots\ldots + n^k$

For any given value of k there is a formula for Σi^k

The first six are

$(k=1)$ $\Sigma i = n(n+1) \div 2$

$(k=2)$ $\Sigma i^2 = n(n+1)(2n+1) \div 6$

$(k=3)$ $\Sigma i^3 = n^2(n+1)^2 \div 4$ *or* $(\Sigma i)^2$

$(k=4)$ $\Sigma i^4 = n(n+1)(2n+1)(3n^2+3n-1) \div 30$

$(k=5)$ $\Sigma i^5 = n^2(n+1)^2(2n^2+2n-1) \div 12$

$(k=6)$ $\Sigma i^6 = n(n+1)(2n+1)(3n^4+6n^3-3n+1) \div 42$

Remainder Theorem

(ax^2+bx+c) divided by $(x-r)$

has a remainder of

ar^2+br+c

(ax^3+bx^2+cx+d) divided by $(x-r)$

has a remainder of

ar^3+br^2+cr+d

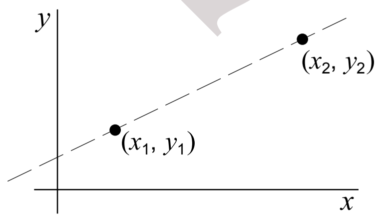
and so on.

Remainder $= 0 \Leftrightarrow (x-r)$ is a factor.

Equation of a Straight Line

Given 2 points (x_1, y_1) and (x_2, y_2) the equation of the straight line joining them is given by $y = mx + c$ where

$m = \dfrac{y_1 - y_2}{x_1 - x_2}$ $c = y_1 - mx_1$



Equation of a Circle

With centre at (a, b) and radius r

Equation is

$(x-a)^2 + (y-b)^2 = r^2$

Permutations

Given n different objects and required to choose r at a time, and the order in which they are chosen DOES matter, this formula gives the number of different ways it can be done.

$^n\mathrm{P}_r = \dfrac{n!}{(n-r)!}$

or

$n(n-1)(n-2)(n-3)\ldots(n-r+1)$

Combinations

Given n different objects and required to choose r at a time, and the order in which they are chosen DOES NOT matter, this formula gives the number of different ways it can be done.

$^n\mathrm{C}_r = \dfrac{n!}{(n-r)!\,r!}$

Fibonacci Sequence

The sequence is generated by

$F_n = F_{n-2} + F_{n-1}$ where $F_2 = 1, F_1 = 1$

from which

$F_3 = 2, F_4 = 3, F_5 = 5, F_6 = 8, F_7 = 13$

(see page 37 for a longer list)

The nth term can found from

$F_n = \dfrac{1}{\sqrt{5}}\left(\dfrac{1+\sqrt{5}}{2}\right)^n$

rounded to the nearest whole number

Taylor's expansion

$$f(x+a) = f(x) + af'(x) + \frac{a^2}{2!}f''(x) + \frac{a^3}{3!}f'''(x) + \dots$$

Maclaurin's expansion

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \dots$$

Trigonometric (*x* in radians)

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots$$

$$\tan x = x + \frac{2x^3}{3!} + \frac{16x^5}{5!} + \frac{272x^7}{7!} + \frac{7936x^9}{9!} + \dots$$

$$\sin^{-1} x = x + \frac{1}{2}\frac{x^3}{3} + \frac{1\times3}{2\times4}\frac{x^5}{5} + \frac{1\times3\times5}{2\times4\times6}\frac{x^7}{7} + \dots \quad (-1 \leqslant x \leqslant 1)$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \dots \quad (-1 \leqslant x \leqslant 1)$$

Hyperbolic

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \frac{x^9}{9!} + \dots = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \frac{x^8}{8!} + \dots = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = x - \frac{2x^3}{3!} + \frac{16x^5}{5!} - \frac{272x^7}{7!} + \frac{7936x^9}{9!} - \dots = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Logarithmic

$$\log_e(1+x) = x - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + \dots \quad (-1 < x \leqslant 1)$$

$$\log_e(1-x) = -x - \frac{x^2}{2!} - \frac{x^3}{3!} - \frac{x^4}{4!} - \dots \quad (-1 \leqslant x < 1)$$

Exponential

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

When the data content is numerical, it is usual to use the symbol x to represent the general case, and individual pieces of data as $x_1\ x_2\ x_3\ x_4\ x_5 \dots x_i \dots\dots x_n$
Another commonly used symbol is **Σ** *(Greek sigma)* which means "find the sum of".
The number of pieces of data is given by n.
If the data is grouped, then f is used to refer to the frequency of the data in each group and that would require a change to some of the formulas given here.
There are also variations to be found in the usage of symbols depending upon whether the reference is to a sample or the whole population.

Standard Deviation

Symbol is σ or s

$$\sigma \text{ or } s = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

Variance is the square of the standard deviation $= \sigma^2$ or s^2

χ^2 (chi-squared) Test

For any group of data, if
O is its **O**bserved frequency
E is its **E**xpected frequency,

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

where the summation is carried out over all the groups of the data.

Rank order correlation coeff.

(Spearman's) gives a measure of the agreement, or otherwise, of two lists of the same items, each ranked in order.
Symbol, r_s

To calculate its value:
Find the difference in value between each corresponding pair of rankings.
Square all the differences, add them together and multiply by 6.
Divide that by $n(n^2 - 1)$, where n is the number of pairs.
Subtract that result from 1.

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

where d is the set of differences between the rankings in the two lists.

Correlation Coefficient

More fully, **Pearson's** product moment correlation coefficient, symbol, r

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{\left[n\sum x^2 - \left(\sum x\right)^2\right]\left[n\sum y^2 - \left(\sum y\right)^2\right]}}$$

$-1 \leqslant r \geqslant 1$ and the closer it is to either, then the better the correlation is.

Line of best fit

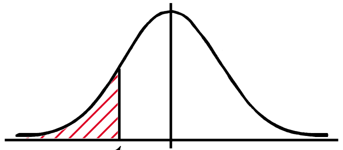
When the data is in the form of ordered pairs of numbers such as (x, y) and there is a good degree of correlation between them *(as determined above)* then it is possible to draw a straight line which can serve as the basis of further calculations.
The equation for this line is of the form

$$y = mx + c$$

The necessary values of m and c can be found from

$$m = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2} \qquad c = \frac{\sum y - m\sum x}{n}$$

Student's *t* distribution



t_p

The table gives t_p for various probabilities p of Student's t distribution having d degrees of freedom. One tail only.

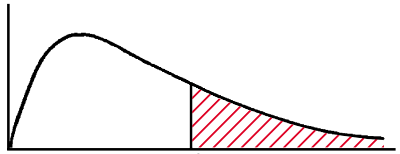
(All rounded to 3 significant figures)

d↓	$t_{0.75}$	$t_{0.80}$	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$	$t_{0.995}$	$t_{0.999}$	$t_{0.9995}$
1	1.00	1.38	3.08	6.31	12.7	31.8	63.7	318	637
2	.816	1.06	1.89	2.92	4.30	6.96	9.92	22.3	31.6
3	.765	.978	1.64	2.35	3.18	4.54	5.84	10.2	12.9
4	.741	.941	1.53	2.13	2.78	3.75	4.60	7.17	8.61
5	.727	.920	1.48	2.02	2.57	3.36	4.03	5.89	6.87
6	.718	.906	1.44	1.94	2.45	3.14	3.71	5.21	5.96
7	.711	.896	1.41	1.89	2.36	3.00	3.50	4.79	5.41
8	.706	.889	1.40	1.86	2.31	2.90	3.36	4.50	5.04
9	.703	.883	1.38	1.83	2.26	2.82	3.25	4.30	4.78
10	.700	.879	1.37	1.81	2.23	2.76	3.17	4.14	4.59
11	.697	.876	1.36	1.80	2.20	2.72	3.11	4.02	4.44
12	.695	.873	1.36	1.78	2.18	2.68	3.05	3.93	4.32
13	.694	.870	1.35	1.77	2.16	2.65	3.01	3.85	4.22
14	.692	.868	1.35	1.76	2.14	2.62	2.98	3.79	4.14
15	.691	.866	1.34	1.75	2.13	2.60	2.95	3.73	4.07
16	.690	.865	1.34	1.75	2.12	2.58	2.92	3.69	4.01
17	.689	.863	1.33	1.74	2.11	2.57	2.90	3.65	3.97
18	.688	.862	1.33	1.73	2.10	2.55	2.88	3.61	3.92
19	.688	.861	1.33	1.73	2.09	2.54	2.86	3.58	3.88
20	.687	.860	1.33	1.72	2.09	2.53	2.85	3.55	3.85
21	.686	.859	1.32	1.72	2.08	2.52	2.83	3.53	3.82
22	.686	.858	1.32	1.72	2.07	2.51	2.82	3.50	3.79
23	.685	.858	1.32	1.71	2.07	2.50	2.81	3.48	3.77
24	.685	.857	1.32	1.71	2.06	2.49	2.80	3.47	3.75
25	.684	.856	1.32	1.71	2.06	2.49	2.79	3.45	3.73
26	.684	.856	1.31	1.71	2.06	2.48	2.78	3.43	3.71
27	.684	.855	1.31	1.70	2.05	2.47	2.77	3.42	3.69
28	.683	.855	1.31	1.70	2.05	2.47	2.76	3.41	3.67
29	.683	.854	1.31	1.70	2.05	2.46	2.76	3.40	3.66
30	.683	.854	1.31	1.70	2.04	2.46	2.75	3.39	3.65
40	.681	.851	1.30	1.68	2.02	2.42	2.70	3.31	3.55
60	.679	.848	1.30	1.67	2.00	2.39	2.66	3.23	3.46
120	.677	.845	1.29	1.66	1.98	2.36	2.62	3.16	3.37
∞	.674	.842	1.28	1.65	1.96	2.33	2.58	3.09	3.29

Chi-squared Distribution

The table gives χ^2_p for various probabilities p of a chi-squared distribution having d degrees of freedom.

(All rounded to 3 significant figures)



χ^2_p

Probabilities

d↓	0.995	.99	.975	.95	.1	.05	.025	.01	.005	.001
1	.0000393	.000157	.000982	.00393	2.71	3.84	5.02	6.63	7.88	10.8
2	.0100	.0201	.0506	.103	4.61	5.99	7.38	9.21	10.6	13.8
3	.0717	.115	.216	.352	6.25	7.81	9.35	11.3	12.8	16.3
4	.207	.297	.484	.711	7.78	9.49	11.1	13.3	14.9	18.5
5	.412	.554	.831	1.15	9.24	11.1	12.8	15.1	16.7	20.5
6	.676	.872	1.24	1.64	10.6	12.6	14.5	16.8	18.5	22.5
7	.989	1.24	1.69	2.17	12.0	14.1	16.0	18.5	20.3	24.3
8	1.34	1.65	2.18	2.73	13.4	15.5	17.5	20.1	22.0	26.1
9	1.73	2.09	2.70	3.33	14.7	16.9	19.0	21.7	23.6	27.9
10	2.16	2.56	3.25	3.94	16.0	18.3	20.5	23.2	25.2	29.6
11	2.60	3.05	3.82	4.57	17.3	19.7	21.9	24.7	26.8	31.3
12	3.07	3.57	4.40	5.23	18.5	21.0	23.3	26.2	28.3	32.9
13	3.57	4.11	5.01	5.89	19.8	22.4	24.7	27.7	29.8	34.5
14	4.07	4.66	5.63	6.57	21.1	23.7	26.1	29.1	31.3	36.1
15	4.60	5.23	6.26	7.26	22.3	25.0	27.5	30.6	32.8	37.7
16	5.14	5.81	6.91	7.96	23.5	26.3	28.8	32.0	34.3	39.3
17	5.70	6.41	7.56	8.67	24.8	27.6	30.2	33.4	35.7	40.8
18	6.26	7.01	8.23	9.39	26.0	28.9	31.5	34.8	37.2	42.3
19	6.84	7.63	8.91	10.1	27.2	30.1	32.9	36.2	38.6	43.8
20	7.43	8.26	9.59	10.9	28.4	31.4	34.2	37.6	40.0	45.3
21	8.03	8.90	10.3	11.6	29.6	32.7	35.5	38.9	41.4	46.8
22	8.64	9.54	11.0	12.3	30.8	33.9	36.8	40.3	42.8	48.3
23	9.26	10.2	11.7	13.1	32.0	35.2	38.1	41.6	44.2	49.7
24	9.89	10.9	12.4	13.8	33.2	36.4	39.4	43.0	45.6	51.2
25	10.5	11.5	13.1	14.6	34.4	37.7	40.6	44.3	46.9	52.6
26	11.2	12.2	13.8	15.4	35.6	38.9	41.9	45.6	48.3	54.1
27	11.8	12.9	14.6	16.2	36.7	40.1	43.2	47.0	49.6	55.5
28	12.5	13.6	15.3	16.9	37.9	41.3	44.5	48.3	51.0	56.9
29	13.1	14.3	16.1	17.7	39.1	42.6	45.7	49.6	52.3	58.3
30	13.8	15.0	16.8	18.5	40.3	43.8	47.0	50.9	53.7	59.7
40	20.7	22.2	24.4	26.5	51.8	55.8	59.3	63.7	66.8	73.4
50	28.0	29.7	32.4	34.8	63.2	67.5	71.4	76.2	79.5	86.7
60	35.5	37.5	40.5	43.2	74.4	79.1	83.3	88.4	92.0	99.6
70	43.3	45.4	48.8	51.7	85.5	90.5	95.0	100	104	112
80	51.2	53.5	57.2	60.4	96.6	102	107	112	116	125
90	59.2	61.8	65.6	69.1	108	113	118	124	128	137
100	67.3	70.1	74.2	77.9	118	124	130	136	140	149

$1 \times 2 \times 3 \times 4 \times \ldots\ldots \times (n-2) \times (n-1) \times n$
is called n factorial

It is shown in symbols as ***n!***

For example
$7! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5040$

0! is defined as 1

$n!$	↓n
1	0
1	1
2	2
6	3
24	4
120	5
720	6
5 040	7
40 320	8
362 880	9
3 628 800	10
39 916 800	11
479 001 600	12
6 227 020 800	13
87 178 291 200	14
1 307 674 368 000	15
20 922 789 888 000	16
355 687 428 096 000	17
6 402 373 705 728 000	18
121 645 100 408 832 000	19
2 432 902 008 176 640 000	20
51 090 942 171 709 440 000	21
1 124 000 727 777 607 680 000	22
25 852 016 738 884 976 640 000	23
620 448 401 733 239 439 360 000	24
15 511 210 043 330 985 984 000 000	25
403 291 461 126 605 635 584 000 000	26
10 888 869 450 418 352 160 768 000 000	27
304 888 344 611 713 860 501 504 000 000	28
8 841 761 993 739 701 954 543 616 000 000	29
265 252 859 812 191 058 636 308 480 000 000	30
8 222 838 654 177 922 817 725 562 880 000 000	31
263 130 836 933 693 530 167 218 012 160 000 000	32
8 683 317 618 811 886 495 518 194 401 280 000 000	33
295 232 799 039 604 140 847 618 609 643 520 000 000	34
10 333 147 966 386 144 929 666 651 337 523 200 000 000	35
371 993 326 789 901 217 467 999 448 150 835 200 000 000	36
13 763 753 091 226 345 046 315 979 581 580 902 400 000 000	37
523 022 617 466 601 111 760 007 224 100 074 291 200 000 000	38
20 397 882 081 197 443 358 640 281 739 902 897 356 800 000 000	39
815 915 283 247 897 734 345 611 269 596 115 894 272 000 000 000	40

2^n	n↓
1	0
2	1
4	2
8	3
16	4
32	5
64	6
128	7
256	8
512	9
1 024	10
2 048	11
4 096	12
8 192	13
16 384	14
32 768	15
65 536	16
131 072	17
262 144	18
524 288	19
1 048 576	20
2 097 152	21
4 194 304	22
8 388 608	23
16 777 216	24
33 554 432	25
67 108 864	26
134 217 728	27
268 435 456	28
536 870 912	29
1 073 741 824	30

2^n	n↓
2 147 483 648	31
4 294 967 296	32
8 589 934 592	33
17 179 869 184	34
34 359 738 368	35
68 719 476 736	36
137 438 953 472	37
274 877 906 944	38
549 755 813 888	39
1 099 511 627 776	40
2 199 023 255 552	41
4 398 046 511 104	42
8 796 093 022 208	43
17 592 186 044 416	44
35 184 372 088 832	45
70 368 744 177 664	46
140 737 488 355 328	47
281 474 976 710 656	48
562 949 953 421 312	49
1 125 899 906 842 624	50
2 251 799 813 685 248	51
4 503 599 627 370 496	52
9 007 199 254 740 992	53
18 014 398 509 481 984	54
36 028 797 018 963 968	55
72 057 594 037 927 936	56
144 115 188 075 855 872	57
288 230 376 151 711 744	58
576 460 752 303 423 488	59
1 152 921 504 606 846 976	60
2 305 843 009 213 693 952	61
4 611 686 018 427 387 904	62
9 223 372 036 854 775 808	63
18 446 744 073 709 551 616	64
36 893 488 147 419 103 232	65
73 786 976 294 838 206 464	66
147 573 952 589 676 412 928	67
295 147 905 179 352 825 856	68
590 295 810 358 705 651 712	69
1 180 591 620 717 411 303 424	70
1 208 925 819 614 629 174 706 176	80
1 237 940 039 285 380 274 899 124 224	90
1 267 650 600 228 229 401 496 703 205 376	100

Systems of Measurement

There are three main systems of measurement still in use. These are known as **imperial units** shown here as (UK); U.S. American units shown as (US); and **metric units** which form the basis of the Système Internationale or SI.

Length (UK) & (US)

12	inches	≡	1 foot
3	feet	≡	1 yard
22	yards	≡	1 chain
10	chains	≡	1 furlong
8	furlongs	≡	1 mile

Area (UK) & (US)

144	sq inches	≡	1 sq foot
9	sq feet	≡	1 sq yard
4840	sq yards	≡	1 acre
640	acres	≡	1 sq mile

Volume (UK) & (US)

1728	cu inches	≡	1 cu foot
27	cu feet	≡	1 cu yard

Capacity (UK) & (US liquid)

4	gills	≡	1 pint
2	pints	≡	1 quart
4	quarts	≡	1 gallon

Capacity (US dry)

2	pints	≡	1 quart
8	quarts	≡	1 peck
4	pecks	≡	1 bushel

Mass (UK) & (US)

437.5	grains	≡	1 ounce
16	ounces	≡	1 pound
14	pounds	≡	1 stone
2000	pounds	≡	1 short ton (US)
2240	pounds	≡	1 long ton (UK)

Length (metric)

10	mm	≡	1 cm
10	cm	≡	1 dm
10	dm	≡	1 metre
10	m	≡	1 dam
10	dam	≡	1 hectometre (hm)
10	hm	≡	1 km

Area (metric)

100	sq mm	≡	1 sq cm
100	sq cm	≡	1 sq dm
100	sq dm	≡	1 sq metre
100	sq m	≡	1 are
100	ares	≡	1 hectare (ha)
100	ha	≡	1 sq km

Volume (metric)

1000	cu mm	≡	1 cu cm
1000	cu cm	≡	1 cu dm (litre)
1000	cu dm	≡	1 cu metre

Mass (metric)

1000	µg	≡	1 mg
1000	mg	≡	1 gram
1000	grams	≡	1 kg
1000	kg	≡	1 tonne

dm is decimetre
dam is dekametre
µg is microgram
mg is milligram

Note (UK) and (US) gallons are NOT the same size *(see below)* so other measures of capacity having the same name are NOT the same size.

Originally every system had its own standard on which its other related measures were based. Now the SI standards are accepted worldwide and all other systems are based on that. These values are **exact** -

1 yard ≡ 0.9144 metres
1 gallon (UK) ≡ 4.546 09 litres
1 pound ≡ 0.453 592 37 kg
1 gallon (US) ≡ 3.785 411 784 litres
1 bushel (US) ≡ 35.239 070 166 88 litres

The old **troy ounce** is still used for dealing in precious metals and stones.

1 troy ounce ≡ 31.103 476 8 grams

Conversion Factors

To change . .	into . .	do this . .
acres	hectares	× 0.4047
centimetres	inches	÷ 2.54 #
cubic cm	cu. inches	× 0.06102
cubic feet	cu. metres	× 0.0283
cubic feet	gallons (UK)	× 6.229
cubic feet	gallons (US)	× 7.481
cubic feet	litres	× 28.32
cubic inches	cu. cm	× 16.39
cubic inches	litres	× 0.01639
cubic metres	cu. feet	× 35.31
feet	metres	× 0.3048 #
gallons (UK)	cu. feet	× 0.1605
gallons (UK)	gallons (US)	× 1.2009
gallons (UK)	litres	× 4.54609 #
gallons (US)	cu. feet	× 0.1337
gallons (US)	gallons (UK)	× 0.8327
gallons (US)	litres	× 3.785
grams	ounces	÷ 28.35
hectares	acres	× 2.471
hectares	sq. miles	÷ 259
inches	centimetres	× 2.54 #
kilograms	pounds	× 2.2046
kilometres	miles	× 0.6214
litres	gallons (UK)	× 0.2200
litres	gallons (US)	× 0.2642
litres	pints (UK)	× 1.760
litres	pints (US liquid)	× 2.113
metres	yards	÷ 0.9144 #
miles	kilometres	× 1.609
ounces	grams	× 28.35
pints (UK)	litres	× 0.5683
pints (UK)	pints (US liquid)	× 1.201
pints (US liquid)	litres	× 0.4732
pints (US liquid)	pints (UK)	× 0.8327
pounds	kilograms	× 0.4536
square cm	sq. inches	× 0.1550
square feet	sq. metres	× 0.0929
square inches	sq. cm	× 6.4516 #
square km	sq. miles	× 0.3861
square metres	acres	÷ 4047
square metres	sq. yards	× 1.196
square miles	hectares	× 259
square miles	sq. km	× 2.590
square yards	sq. metres	÷ 1.196
yards	metres	× 0.9144 #

# indicates an **exact** figure, all others are approximations.

The Greek Alphabet

The Greek alphabet is a rich source of symbols used in both mathematics and science, to the extent that nearly every one of them (both capitals and lower case) is used in some way or other. Some of them appear more than once to represent different things. Below is the full alphabet, and the names of the various symbols. The capital form of the letter is given in the first column, followed by the lower case version and its name. Some of the more common usages are given.

Α	α	alpha	α β γ are often used to identify angles in plane figures.
Β	β	beta	
Γ	γ	gamma	
Δ	δ	delta	Δ is sometimes used to represent the area of a plane figure. δ is used (in calculus) to show a small change.
Ε	ε	epsilon	ε is often used for a small number.
Ζ	ζ	zeta	
Η	η	eta	
Θ	θ	theta	θ is often used to indicate a general angle.
Ι	ι	iota	
Κ	κ	kappa	
Λ	λ	lambda	λ is used to represent a scalar in vector work.
Μ	μ	mu	μ is used (in the SI system) to represent the prefix *micro*. *μ* is sometimes used to represent the arithmetic mean.
Ν	ν	nu	
Ξ	ξ	xi	
Ο	ο	omicron	
Π	π	pi	Π is used to show that a continued product is needed. π represents the irrational number 3.14159 … π(*n*) means the number of primes less than, or equal to *n*.
Ρ	ρ	rho	
Σ	σ	sigma	Σ is used to show that the sum of a set of numbers is to be found. σ is used to represent the standard deviation.
Τ	τ	tau	τ used to represent the golden ratio 1.6180 … (see also phi).
Υ	υ	upsilon	
Φ	φ	phi	*ϕ* used to represent the golden ratio 1.6180 … (see also tau). *ϕ* (*n*) is number of positive integers less than, and relatively prime to, *n*.
Χ	χ	chi	χ is used in statistics in reference to the chi-squared test.
Ψ	ψ	psi	
Ω	ω	omega	ω is used for an angular speed.

Symbols and Abbreviations

continued from inside Front Cover

	the universal set
∈	is a member of
∉	is not a member of
⊂	is a subset of
⊄	is not a subset of
⊃	includes
∪	union
∩	intersection
∅	null *or* empty set
i	square root of −1
e	≈ 2.71828 …
π	≈ 3.14159 …
f(*x*)	function of *x*
f′(*x*)	first derivative of f(*x*)
f″(*x*)	second derivative of f(*x*) and so on
$\int$	integral *or* anti-derivative
$\int f(x)dx$	indefinite integral
$\int_a^b f(x)dx$	definite integral
Σ	summation
&	hexadecimal number follows
i, **j**, **k**	unit vectors
m	gradient of a line
|A|	determinant of matrix A
a|*b*	*a* is exactly divisible by *b*

AER	annual equivalent rate
AP	arithmetic progression
APR	annualised percentage rate
cu	cubic *(referring to units of volume)*
dp	decimal places
gcd	greatest common divisor
GM	geometric mean
hcf	highest common factor
iff	if and only if
lcd	lowest common denominator
lcm	lowest common multiple
ln	logarithm, natural *also written* $\log_e$
mod	modulus
QED	which was to be proved
RMS	root mean square
sf	significant figures
sq	square *(referring to units of area)*
UT	Universal Time *(Greenwich Mean Time)*

sin	sine
cos	cosine
tan	tangent
sec	secant
cosec	cosecant
cot	cotangent
arcsin	$\sin^{-1}$
arccos	$\cos^{-1}$
arctan	$\tan^{-1}$
vers	versine
covers	coversine
hav	haversine
sinh	hyperbolic sine
cosh	hyperbolic cosine
tanh	hyperbolic tangent